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ON THE MAGNETIC PROPERTIES OF HEUSLER'S ALLOYS.

By J. C. McLennan.

SINCE the discovery by Heusler some three years ago that it was possible to prepare alloys of copper, manganese, and aluminium which possessed magnetic properties similar to and comparable in amount with those of cast iron, considerable attention has been paid by a number of investigators[1] to this interesting phenomenon.

Among others Mr. H. F. Dawes, M.A., and Miss L. B. Johnson, M.A., demonstrators in the Physical Laboratory at Toronto, and Messrs. H. A. McTaggart and J. K. Robertson, senior students in the same laboratory have from time to time during the past year, as opportunity offered, studied the properties of some of these alloys which they prepared, and the following paper contains a few notes on the results of their work, which is still being continued by two of them.

In order to prepare samples of the alloys a supply of manganese, copper, and aluminium, sold as chemically pure and free from iron was obtained from Messrs. Eimer and Amend of New York. The

[1] Fr. Heusler, W. Starck and E. Haupt, Verh. der Phys. Ges., 5, p. 219, 1903. Heusler, Starck and Haupt über der ferromagnetischen Aigenschaften von Legierung unmagnetischer Metalle, Marburg, 1904. Gumlick, Ann der Physik, 16, p. 535, 1905. Austin, Verh. der Deut. Phys. Gesell., 6, p. 211, 1904. Fleming and Hadfield, Proc. Roy. Soc., 76, p. 271, 1905. E. Take, Verh. der Deut. Phys. Gesell., 7, p. 133, 1905, Ann. der Physik, Vierte Folge, Band 20, p. 249. Hill, PHYS. REV., 21, p. 335, 1905; 23, p. 498, 1906. Guthe and Austin, Bulletin of the Bureau of Standards, Washington, Vol. 2, No. 2, page 297, Aug., 1906.

metals were melted in an ordinary coke furnace, such as is used in brass foundries, and poured into moulds just as in making brass castings. A graphite crucible was used for the purpose, and in the process the copper was first heated until it reached a molten state, after which the manganese was added to it. When this in turn was melted the aluminium was added, and the molten mass then stirred for a short time with a porcelain rod, after which the crucible was taken from the furnace and the melted metal poured into the moulds.

For a preliminary set of the alloys four castings were made. As Heusler had found that the most highly magnetic alloy contained manganese and aluminium in the proportion of their atomic weights, one casting was made with as nearly as possible this proportion of the two metals. A second was made with a larger proportion of aluminium and two others with smaller proportions of this constituent.

The percentages of the metals used in each of the four castings and the proportions of aluminium to manganese by weight and by atomic ratios are shown in Table I.

TABLE I.

Number of Alloy.	Composition of Alloys — Percentages.			Ratio of Manganese to Aluminium by Weight.	Atomic Ratio of Aluminium to Manganese.
	Aluminium.	Manganese.	Copper.		
I.	8.0	32.1	59.8	.250	.51
II.	9.7	25.6	64.6	.378	.77
III.	14.3	28.6	57.1	.498	1.01
IV.	15.9	23.9	60.3	.683	1.392

In each casting sufficient material was used to make two cylindrical rods about 20 cm. long, and 8 mm. in diameter, as well as two rings about 7 cm. in diameter, and 2.5 cm. in height. It was thought desirable to make the castings in the form of rings as well as rods, in order to investigate more fully the permeability of each specimen.

In the whole of the operation the greatest care was taken to keep the alloy free from iron in order that there might be none of this metal present to vitiate the results or to cast doubt upon their validity.

The investigations described below include one by Mr. Dawes on the phenomenon of magnetostriction exhibited by one set of the rods, one by Messrs. McTaggart and Robertson on the same phenomenon with the second set of rods, and one by Miss Johnson on the permeability of the alloys cast in the form of rings.

A. Magnetostriction. (Experiments by Mr. Dawes.)

I. *Apparatus.* — The apparatus used in measuring the elongation was, with some changes, modelled after that used by Rhoads[1] in his investigations on magnetostriction in iron. The apparatus is shown in section in the accompanying diagram, Fig. 1. A long hollow solenoid L produced the magnetic field in which the rod to be tested was placed. The lower end of the rod F was screwed into the fixed brass stand H, and the upper end carried a brass rod E. A cylindrical water jacket K placed between the coil and the rod served to keep the latter at a constant temperature. The change of length of the rod was magnified by means of an optical lever. A brass block D carrying a brass pivot indicated by the inverted arrow was clamped to the rod E, and so moved up or down as the rod F was elongated or shortened. A brass tube G which was attached to H at the base of F supported a horizontal projection A. A carried two brass pivots indicated by the second arrow. A lever B rested horizontally between these sets of pivots as indicated. At C a mirror was mounted so that it could rotate about an axis perpendicular to the plane of the paper and attached to A. This mirror was supported in a vertical plane by an attachment resting on the end of B. The brass points rested on agate beds and the axis was jewelled. The image of a scale as seen in C was

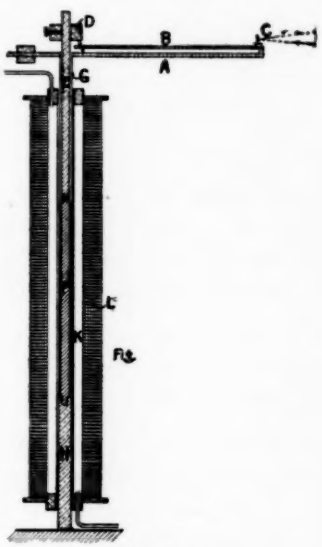


Fig. 1.

[1] Rhoads, Phys. Rev., Vol. VII., No. 2, p. 65, 1898.

observed by means of a telescope. In order to trace the method by which the change of length was measured, suppose that the rod was slightly elongated relative to the tube G. The brass point carried by D, would then move upwards, and in consequence the C-end of the lever B would be allowed to drop. This would cause the mirror to be tilted backwards, and hence cause a change in the scale reading as observed by the telescope. It will be seen that if the change of length was very small, it would be proportional to the change of scale reading, and the proportionality factor could be found by measuring the ratio of the arms of the lever, the distance between the axis of rotation of the mirror and its point of support, and the distance of the scale from the mirror.

In the present experiment a modification was made in the apparatus as used by Rhoads. Formerly a plane mirror was used at C and the scale was placed at a distance of about 4 meters from it. This mirror was replaced in these experiments by a concave mirror of one meter radius of curvature, and the scale was placed at a distance from it of one half meter, the focal length of the mirror. By this means the image of the scale was magnified sixteen times its former size and the sensibility, *i. e.*, the change of reading for a given change of length was double that obtained with the plane mirror. A glass scale was used in the telescope instead of a cross hair, and the number of divisions on this scale over which a line of the image moved was taken as the reading. The sensibility of the arrangement was such that a change of reading of one division resulted from a change in length of the specimen of 10^{-5} mm.

II. *Measurements.* — In making measurements the change of scale reading was found for a series of applied fields ranging from about 30 to about 900 C.G.S. units. The strength of the applied field was calculated from the value of the current, and the known dimensions of the coil and the change of length per centimeter was deduced from the scale readings. For each series of readings curves were plotted, the strengths of field, H, being taken as abscissæ, and the changes per unit length multiplied by 10^7 as ordinates.

In the first few series of measurements, the readings were taken by applying each field for an instant and then withdrawing it.

By this procedure, an average result was obtained for the effect

investigated, but the method was not suitable for bringing out any hysteresis effect which might be present. It was therefore discarded, and the method of increasing the field step by step and then reducing it in the same way was adopted.

It was found that a cycle of readings could be taken in this way in about two minutes.

When the current was kept passing through the coil it might be expected that the heating would effect the change of length. In order to test for errors from this cause, a brass rod was inserted in the apparatus in place of one of the specimens investigated, and a cycle of fields similar to that used in working with the alloys applied to it. As brass is non-magnetic and consequently does not exhibit the phenomenon of magnetostriction, the development of heat by the current and its diffusion into the walls of the water jacket would be followed by the elongation of *G*, Fig. 1, which would produce an apparent shortening of *D*. This test showed that during the time of a cycle of observations no error from heating was introduced.

Elongation curves for each of the four specimens are shown in Figs. 2–5, the corresponding sets of values being given in Table II. Curves 1 and 3 *A* were taken by the first method, the others by the method of cycles.

TABLE II.

Alloy No. 1.

H	$\frac{dl}{l} \cdot 10^7$	H	$\frac{dl}{l} \cdot 10^7$
33	0	220	.3
50	0	900	.5
130	.1		

Alloy No. 2.

33	0	495	− .5
95	.15	375	− .75
225	.25	335	−1.0
335	.25	286	−1.0
375	.25	225	−1.25
491	−.25	95	−1.5
	−.5		
945	−.25		

Alloy No. 3. Curve A.

22	.25	315	3.8
44	.5	352	4
67	.6	385	4.2
90	.8	435	4.5
124	1.0	456	5.5
192	2.0	980	9
255	2.3		
276	2.5		

Alloy No. 3. Curve B.

33	.5	470	32
56	1.4	385	31
79	4.4	351	30.4
100	8	310	30
143	15	260	29
215	20	215	27.5
260	23.2	143	22.8
310	26	100	19
351	27	79	14
385	27.4	56	9.5
470	27.5	33	5.8
910	33	0	3

Alloy No. 4. Curve A.

33	.5	912	3.2
56	1	475	3
85	1.5	375	3
110	2	357	2.5
143	2.4	320	2.3
220	2.5	275	1.5
275	2.5	221	1
320	2.5	143	.5
385	2.5	110	0
475	2.6	85	−2.5

Alloy No. 4. Curve B.

33	2	490	11.5
56	5.5	400	11.5
90	7.25	332	11.5
111	8.5	221	11
170	9.5	170	11
221	11	110	11
332	11.5	90	10.5
480	11.5	56	10
940	23	33	16
		0	3

III. *Discussion.* — A careful examination of these and the other curves found in the series of measurements showed that three factors entered into the change of length of the bar.

1. When a change was made in the field to which the bar was subjected an almost instantaneous change was made in the length. This change consisted in a lengthening as the field was increased, and a shortening as it was decreased.

2. As the field was decreased there was a certain lag or hysteresis effect so that the length did not have the same value for a given field as the field was approached from above or below.

3. In some cases when the specimen was left in a field for some time there was a continuous shortening. This effect when it existed was very marked in strong fields so that the time for which the specimen was left in these fields had a comparatively great influence on the length. This effect was also observed by Austin in some experiments made by him with these alloys. It does not appear, however, to have been found by him in some measurements he has recently made, in collaboration with Guthe, with other samples of the alloys.

The form of the curve arising from a cycle of readings on a given specimen would depend, therefore, on the relative importance of

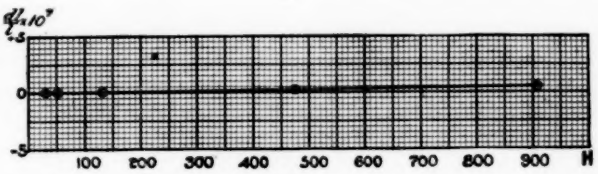


Fig. 2.

these three factors. In the early stages of an increasing field there would be no effect from (2) and that from (3) would be small so that the initial portions of the elongation curve would correspond to the change of length arising from the first source. In each of the curves shown, this part of the curve exhibits the same character. The curve rises slowly in the very weak fields and then in fields extending from about 50 to 200 units quite rapidly. In still higher fields the rise becomes smaller and tends to a maximum value.

In the case of alloy No. 2, Fig. 3, it will be seen that while the

actual changes were small compared with those in alloys No. 3 and No. 4, the third factor was of more importance than the first, with

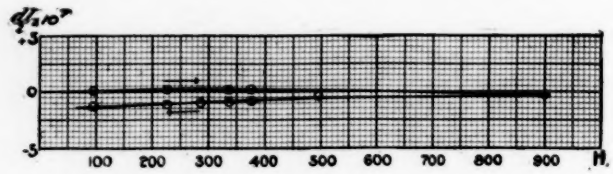


Fig. 3.

the result that in the higher fields the bar diminished a little below its original length. This was also observed in the case of alloy No. 1, although it is not shown in the curve as the experimenter failed to keep a record of his measurements.

In curves 3, *B* and 4, *B*, Figs. 4 and 5, this cause of change of

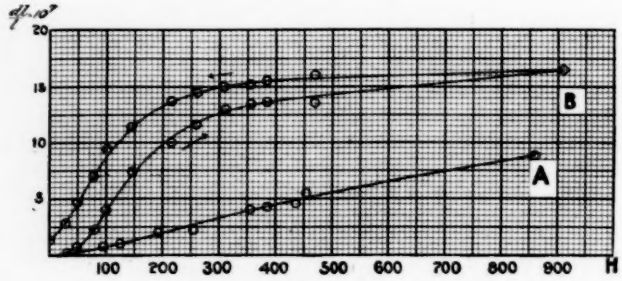


Fig. 4.

length was not sufficiently great to make the return half of the curve cut the forward half, *i. e.*, to overcome the hysteresis effect. But if the strong fields had been left on a little longer this would probably

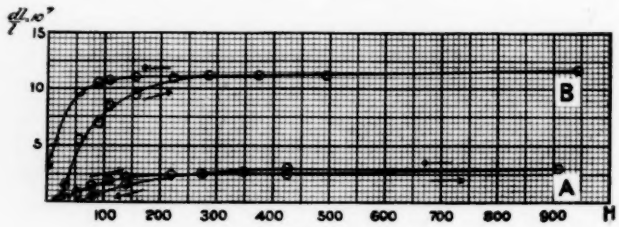


Fig. 5.

have been the case and there would have been a final resultant contraction. An example of this effect is shown in 4, *A*, Fig. 5, where

the return portion of the curve crosses the forward initial portion and consequently masks the presence of the hysteresis effect shown in 4, *B*.

In Figs 4 and 5, curves *A* represent the elongation obtained with two of the alloys when magnetized for the first time, and curves *B*, in the same figures represent the elongations obtained with the same rods after they had been magnetized and demagnetized several times, and then allowed to lie undisturbed for about thirty hours.

This increase in the elongation has not hitherto been observed, and it may possibly be connected with the increase of permeability observed by Starck and Haupt,[1] Gumlich,[2] Hill[3] and others with these alloys when they were heated for some time to a moderate temperature, such as a 110° C. after being cast. It may possibly, too, point to changes in the constitution of the alloy analogous to those marking the recovery of iron from overstrain observed by Muir,[4] on heating strained iron bars for short periods in boiling water.

Considering the curves as a whole it will be seen that the elongation increased with the percentage of aluminium, present in the alloy, the maximum elongation, curve *B*, Fig. 4 being obtained with the alloy in which the manganese and aluminium were present in the proportions of their atomic weights.

The maximum elongation in this case, it will be seen, was 16 ten-millionths or about one third the maximum for iron.

The effect of higher percentages of aluminium is shown in Fig. 5, curve *B*, where the maximum elongation was 11.75 ten-millionths.

Chemical analyses of the alloys have not as yet been made, but if the constituents of the different samples of the alloys can be taken to have the same relative proportion as the metals from which they were cast, these results would go to show that the maximum elongation was obtained with alloys containing the same proportions of manganese and aluminium as were found by Heusler to give the highest permeability.

[1] Deutsch. Phys. Gesell. Verh., 5, 12, pp. 224-232, June, 1903.
[2] Electrotech. Zeit., XXVI., 9, Mar., 1905.
[3] Hill, PHYS. REV., Vol. XXIII., No. 6, p. 498, 1906.
[4] Phil. Trans. Roy. Soc., London, Vol. 193, pp. 1-46, Series A.

B. MAGNETOSTRICTION. (EXPERIMENTS BY MESSRS. H. A. MCTAGGART AND J. K. ROBERTSON.)

In the experiments made with the first set of rods a very marked difference was observed in the elongations obtained with alloys Nos. 3 and 4, when magnetized for the first time, from what was obtained with them after they had been magnetized and demagnetized several times. This result pointed to the existence of an unstable equilibrium among the molecules of newly cast specimens of the alloys, and it forms a corroboration of the opinions of other experimenters who have arrived at the same conclusions from their observations on the changes in permeability of the alloys produced by variations in temperature.

TABLE III.

H	$\frac{dl}{l} \cdot 10^7$	H	$\frac{dl}{l} \cdot 10^7$
Alloy No. 2.			
235	0.8	300½	−0.8
370	1.7	218	−1.7
531	2.5	155	−2.5
675	2.5	0	−3.3
440	0		
Alloy No. 3.			
55	1.6	580	13.3
96	3.3	376	12.5
130	4.2	255	10.8
198	6.6	202	10
255	8.3	155	8.3
272	9.2	118	6.6
376	10.8	75	4.2
474	12.5	0	−1.6
Alloy No 4.			
32	4.2	255	15.8
62	8.3	180	15
96	10.8	125	14.2
145	12.5	84	12.5
202	14.2	55	10
250	15	40	8.3
355	15.8	10	4.2
474	16.7	0	0
595	17.1		

It seemed desirable to investigate this point more closely, and therefore a set of measurements was made by the above mentioned senior students on the elongations of the second set of rods cast at the same time as those used by Mr. Dawes. Before proceeding with this however, a set of readings was taken with the rods used by the latter to see whether any change had taken place in them in the six months interval during which they had been laid aside. The results of their measurements are given in Table III. and represented graphically by the curves in Figs. 6–8.

With alloy No. 1 no elongation whatever could be observed by them. With alloy No. 2, Fig. 6, the general form of the curve they obtained is similar to that given in Fig. 3, by Mr. Dawes, although the maximum elongation they obtained in increasing fields and the maximum shortening in decreasing fields was greater than that observed by him with this specimen.

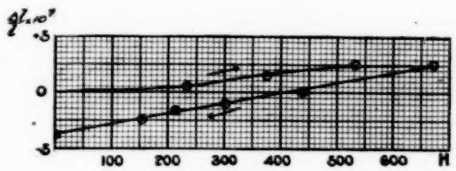


Fig. 6.

Their results with alloy No. 3, Fig. 7 show the presence of the three effects pointed out in the first set of measurements, the hysteresis and the shortening being clearly marked. The maximum

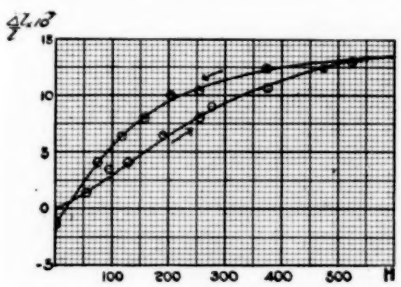


Fig. 7.

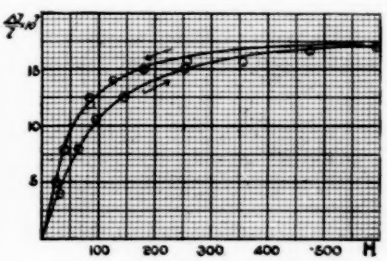


Fig. 8.

elongation obtained by them with this specimen corresponded to a field of 600 units and is slightly less than that obtained in the first tests for the same field.

With alloy No. 4, Fig. 8, the elongation was about 50 per cent. greater than that shown in Fig. 5. In both curves there exists a

well marked lag, but in the later measurements, the rod returned to its original length whereas in the earlier ones it showed an elongation when the field was removed.

It is evident from these measurements that considerable modifications must have taken place in the structure of the alloy in the interval which elapsed between the two tests, and that the lapse of time is a factor as well as changes in temperature and magnetization in producing alterations in the molecular groupings in the alloy.

TABLE IV.

Alloy No. $2A_1$.

H	$\frac{dl}{l} \cdot 10^7$	H	$\frac{dl}{l} \cdot 10^7$
0	3.3	445	2.9
174	3.3	330	2.9
425	3.75	284	2.5
745	4.1	180	1.7
670	3.75	104	0.8
635	3.3		

Alloy No. $2A_2$.

118	0.8		
280	1.7		
705	2.1		
425	1.7		
155	0.8		
0	0		

Alloy No. $3A_1$.

42	2.5	910	27.5
86	8.3	600	25
138	12.5	405	23.3
180	15	320	21.7
215	16.7	280	20.8
312	20	215	19.2
390	21.7	121	15
460	22.5	75	8.3
545	24.2	42	5.8
705	25	0	4.2
890	26.7		

TABLE IV.—*Continued.*

Alloy No. $3A_2$.

50	2.5	705	20.8
75	4.2	880	22.5
104	6.7	635	20.8
125	8.3	405	19.2
155	10	270	16.7
185	12.5	175	12.5
238	14.2	104	8.3
320	16.7	60	4.2
398	18.3	0	0
495	19.2		

Alloy No. $3B_1$.

53	1.7	740	23.3
80	4.2	592	22.5
110	5.8	425	20.8
138	8.3	370	20
192	12.5	326	19.2
225	14.2	284	17.5
277	16.7	255	16.7
348	18.3	208	15
372	19.2	174	12.5
458	20.8	118	8.3
565	21.7	68	4.2
610	22.5	0	1.7

Alloy No. $3B_2$.

55	1.7	545	21.7
85	4.2	390	20.8
134	8.3	348	20
187	12.5	327	18.3
275	16.7	255	16.7
390	19.2	187	12.5
480	20.8	125	8.3
665	22.5	68	4.2
760	23.3	0	0

Alloy No. $4A_1$.

112	3.3	565	7.5
208	4.2	452	7.5
270	5	320	7.5
362	5.8	215	7.5
390	6.7	103	6.7
478	6.7	0	5
705	7.5		

TABLE IV.—*Continued.*

Alloy No. 4A₂.

50	1.7	362	2.9
110	2.5	255	2.5
362	3.3	118	2.5
552	3.4	70	2.1
610	3.4	42	1.25
590	2.9	0	0

The measurements made with the duplicate set of rods are given in Table IV., and curves representing them are given in Figs. 9, 10, 11, 12, 13, and 14. For convenience in description the rods used in this investigation are designated as No. 1, *A*, No. 2, *A*, No. 3, *A*, and No. 4, *A*, and where subscript figures are used, for example in No. 3, A_1, and in No. 3, A_2 they represent the first and second tests respectively made with the specimen. Alloy No. 3, *B* is a rod of another casting made over a year ago from the same metals and with the same proportions as the pair No. 3 and No. 3, *A*.

With No. 1, *A* no elongation was observed which accorded exactly with the behavior of the twin rod No. 1.

With No. 2, *A*, Fig. 9, the results obtained were very similar to those obtained with rod No. 2. Curve No. 2, A_1 represents the elongations obtained with the first magnetization. As the field was

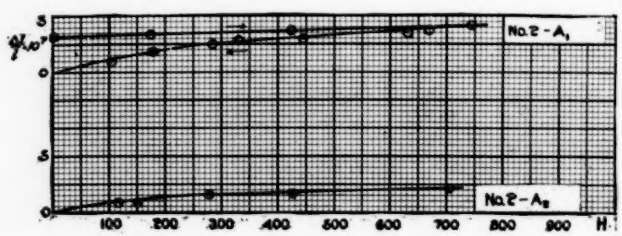


Fig. 9.

gradually increased to 750 units the rod steadily lengthened, and as the field was decreased the rod shortened, and finally when the field was removed entirely it remained slightly shorter than its original length. It was then repeatedly magnetized and demagnetized and afterwards put through the usual cycle of fields. The curve No. 2, A_2 represents its behavior after this treatment. The

rod as will be seen had assumed a steady state, and the upward and downward curves coincided exactly. No hysteresis action or shortening under high fields was observed.

With alloy No. 3, *A* the measurements made with the first magnetization are given in Fig. 10. Here it will be seen a lag was

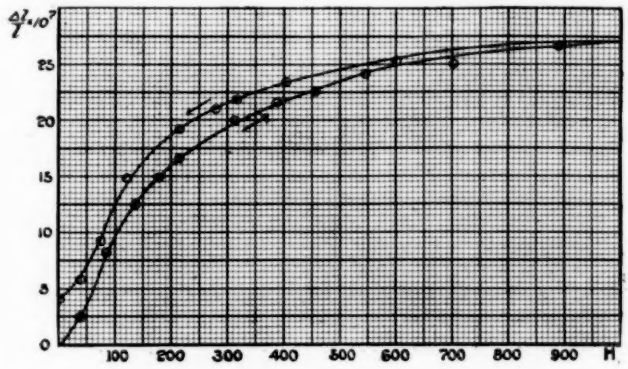


Fig. 10.

obtained in the return portion of the cycle, the rod ending with a slightly greater length than it had originally.

Curve No. 3, A_2, Fig. 11, shows its behavior after being repeatedly magnetized. Here again a lag was observed in the return

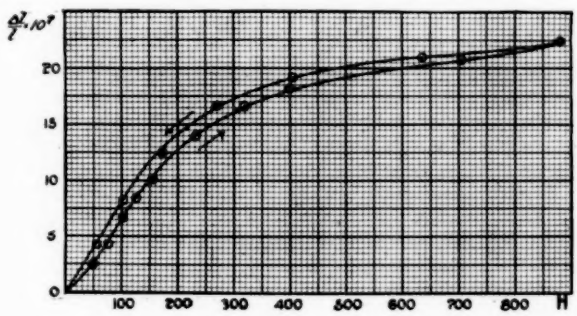


Fig. 11.

portion of the cycle, but the shortening referred to under heading No. 3 page 455 had entirely disappeared, so that the initial and final lengths of the rod were the same.

The first and second tests with alloy No. 3, *B* are shown in Figs. 12 and 13. Here again it will be seen, precisely the same behavior

was obtained as with No. 3, *A*. As with the latter a lag was again observed in the return portion of the cycle and a final lengthening of the specimen occurred. In the second tests, Fig 13, made after

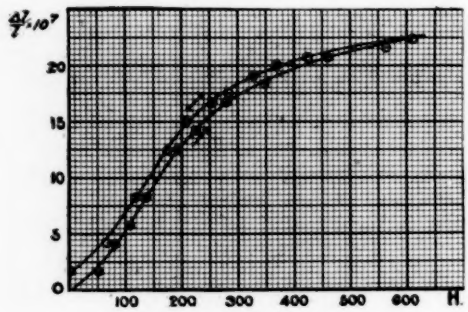


Fig. 12.

repeated magnetizations, this change of length disappeared, and the rod after being put through its cycle returned to its original length.

With alloy No. 4, *A*, the behavior was exactly the opposite of that obtained with No. 2, *A*. In the initial test, No. 4, A_1, Fig. 14, the rod steadily lengthened as the field was increased to 700 units

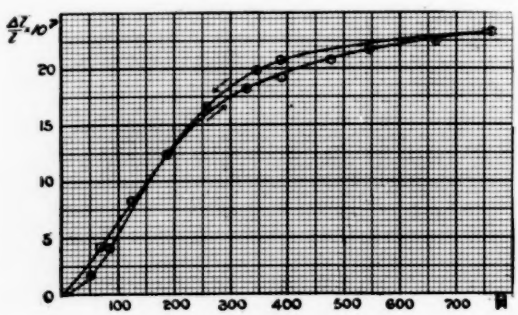


Fig. 13.

and afterwards as the field was decreased a lag occurred and the rod ended somewhat longer than it was at the commencement of the cycle. After it had been repeatedly magnetized and demagnetized it was again put through a cycle, and curve No. 4, A_2 was obtained. Here again the values in the downward portion of the cycle coincided exactly with those obtained in the forward portion and no lag and no shortening was observed.

In the measurements with these rods, the greatest elongation was obtained with samples No. 3, *A* and No. 3, *B*. With them the maximum elongation observed was about 22.5 ten-millionths which is somewhat greater than that obtained with the specimen No. 3 where the maximum elongation obtained was 16 ten-millionths.

The maximum elongation obtained with rod No. 4, *A* was somewhat less than that obtained by Mr. Dawes with rod No. 4. With the former specimen the maximum elongation was about 3 ten-millionths while that obtained with the latter was over three times as great. This discrepancy was very likely due to a difference in the composition of the two rods which should be brought out by a chemical analysis of the specimens. It is interesting to note that with both sets of rods the maximum elongation was obtained when the composition of the alloy was approximately the same as that of the specimens which were found by Heusler to give the highest permeability.

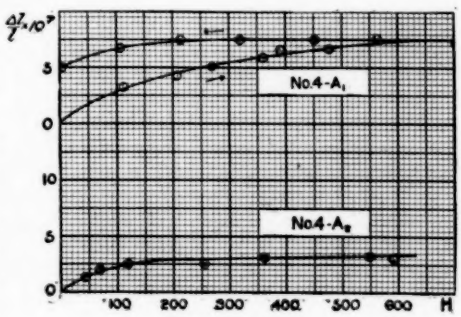


Fig. 14.

Summarizing the results of the two measurements it would appear that the structure of the alloys when freshly cast is very unstable, and that it undergoes a rapid change under repeated magnetizations and a slower change under the lapse of time.

In their experiments on magnetostriction Guthe, and Austin state that no after-effect either in expansion or in contraction was observed even with the strongest fields, whereas Austin in his first communication refers very definitely to the observation of such an effect in his measurements. The explanation probably is that in the first investigations the rods were in the same condition as those with which the initial tests described in this paper were made; while in the later investigation the rods tested (which were obtained from Dr. Heusler) had very likely been subjected to such conditions prior to the test as to bring them into the stable condition.

III. PERMEABILITY. (EXPERIMENTS BY MISS L. B. JOHNSON.)

During the period covered by the experiments on magnetostriction, a series of measurements was made by Miss L. B. Johnson on the permeability of the four alloys. Two rings had been made at each casting, in addition to the two rods mentioned above, and with these the experiments on permeability were carried out. The magnetization was measured by the ballistic method, by reversing the magnetizing force, and the arrangement of apparatus was similar to that used by Ewing and Klaasson.[1]

TABLE V.

Designation of Ring.	Height.	Thickness.	Mean Diam.	Cross Section.	Turns in Primary.	Turns in Secondary.
No. I.	1.56 cm.	.881 cm.	6.59 cm.	1.374 sq.cm.	155	100
No. II.	1.47 "	.855 "	6.615 "	1.256 " "	153	100
No. III*a*.	2.44 "	.623 "	6.98 "	1.516 " "	150	50
No. III*b*.	2.37 "	.68 "	6.87 "	1.61 " "	115	50
No. IV*a*.	2.39 "	.703 "	6.86 "	1.684 " "	157	50
No. IV*b*.	1.447 "	.838 "	6.64 "	1.212 " "	136	46

TABLE VI.

Alloy No. I.

H	*B*	μ	*H*	*B*	μ
1.39	1.39	1.0	14.40	18.7	1.3
2.82	4.01	1.4	28.22	37.8	1.3
3.93	5.35	1.3	37.63	50.8	1.3
7.52	10.7	1.4	86.45	78.9	1.3

TABLE VII.

Alloy No. II.

H	*B*	μ	*H*	*B*	μ
.814	1.464	1.7	17.5	27.8	1.5
1.66	2.92	1.7	42.5	68.8	1.6
4.99	8.78	1.7	69.3	106.8	1.5
8.55	13.17	1.5			

[1] Phil. Trans., Vol. CLXXXIV., A, p. 987, 1893.

TABLE VIII.

Alloy No. IIIa.

H	*B*	μ	*H*	*B*	μ
.68	6.442	9.3	17.1	201	11.6
1.8	17.1	9.8	28.3	332.8	11.7
4.4	46.1	10.3	36.9	364	9.8
7	71.2	10.1	42.9	413.8	9.6
11.9	129.8	10.8	59.3	483.1	8.1

The dimensions of the rings which have been designated as I., II., III*a*, III*b*, IV*a*, IV*b* for the purpose of identifying them with the rods which were denoted by the same numbers, are given in Table V. The table also contains the number of turns in the primary and secondary coils of each ring.

TABLE IX.

Alloy No. IIIb.

H	*B*	μ	*H*	*B*	μ
.99	4.596	4.6	11.24	70.17	6.2
1.45	9.138	6.3	23.4	160.8	6.8
2.04	11.42	5.5	30.81	221.5	7.1
3.08	18.27	5.9	50.75	352.1	6.9
6.01	34.26	5.7			

The first set of measurements was made with the rings, just as they were cast, and the results are given in Tables VI., VII, VIII., IX., X., XI. The permeability curves are given in Fig. 15.

TABLE X.

Alloy No. IVa.

H	*B*	μ	*H*	*B*	μ
.2745	13.1	48.1	8.692	565.7	65.1
.5032	30.5	58	16.47	933.9	56.7
1.464	96.1	65.6	34.31	1487.2	43.3
2.379	157.2	66.1	45.29	1724.08	38
4.575	314.5	68.7	69.54	2072.3	29.8

TABLE XI.

Alloy No. IVb.

H	*B*	μ	*H*	*B*	μ
.0379	3.108	82	16.45	1366.93	83.0
.147	13.67	92.9	27.14	2113.44	77.8
.452	49.72	110	34.55	2257.76	65.4
1.192	136.75	114.7	41.77	2424.24	58
2.467	279.72	113.3	51.00	2548.4	49.8
3.331	360.52	108.2	55.93	2586.4	44.4
5.264	535.81	101.7	80.61	2735.04	33.8
7.81	739.70	94.7	92.13	2735.04	24.6

From these it will be seen that Alloys IV*a* and IV*b* showed the highest permeability. Alloys I. and II. were but feebly magnetic, and Alloys III*a* and III*b* only slightly so.

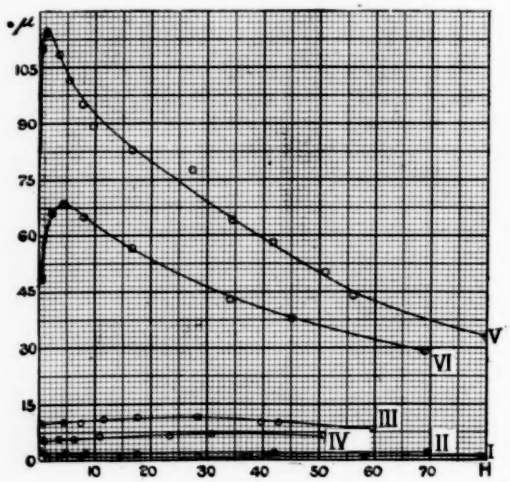


Fig. 15.

It had been expected that the permeabilities of the rings would give an indication of the relative permeabilities of the corresponding rods used in the measurements on magnetostriction. The very considerable difference in the permeability of the two rings No. IV*a* and No. IV*b*, which were made at the same casting, showed, however, that even different portions of the same casting did not exhibit the same magnetic properties, and that before any definite comparisons could be made, it was necessary to make a chemical

analysis of each of the specimens tested. This difference in magnetic properties was also very marked with alloy No. III. The rods of this casting gave the greatest elongation, while the rings, as the figures show, possessed but a small permeability. Guthe and Austin[1] have pointed out that the greatest elongation was obtained by them with rods which possessed the greatest permeability and as some experiments which are now being made on the permeability of the rods No. III. show that they possess a very much higher permeability than that exhibited by the rings, this conclusion is very probably correct.

TABLE XII.

Alloy No. IIIa.

H	*B*	μ	*H*	*B*	μ
.73	8.59	11.9	17.1	309.2	17.9
1.28	15.03	11.7	27.5	440.1	15.9
1.97	25.77	13	36.1	541.1	14.9
6.4	94.4	14.7	46.4	674.3	14.5
6.8	105.2	15.3	64.5	751.6	11.6
8.5	138.2	16.2	91.1	843.9	9.2
11.5	196.5	17			

Alloy No. IVa.

H	*B*	μ	*H*	*B*	μ
.164	9.83	59.9	8.88	758.02	85.3
.814	65.33	80.5	16.4	1187.42	72.4
1.41	121.02	85.8	34.31	1857.5	54.1
2.33	203.15	87.1	43	2090.1	48.8
4.3	415.05	96.7	71.32	2402.6	33.6
6.03	538.26	89.2			

Alloy No. IVb.

H	*B*	μ	*H*	*B*	μ
.7621	10.567	13.9	24.38	528.22	21.6
1.258	28.172	16.3	31.24	654.99	20.9
3.35	56.34	16.4	44.2	795.85	18
6.09	116.2	19	58.68	993.06	16.9
9.9	211.29	21.3	86.11	1253.6	14.55
15.24	352.15	23.1			

[1] Loc. cit.

In addition to the initial tests just described, some measurements were also made on the changes in the permeability produced by continued heating.

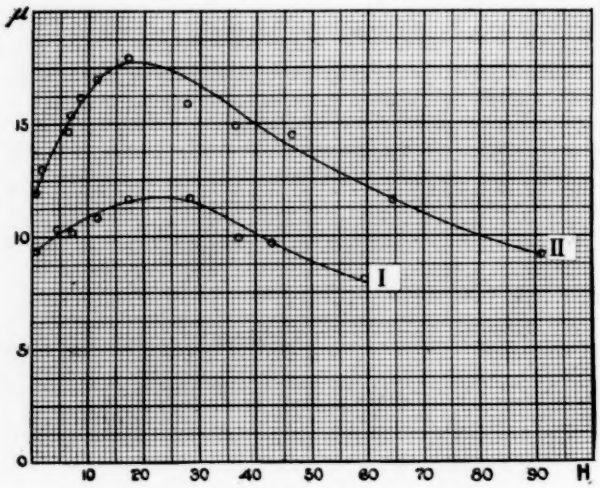


Fig. 16.

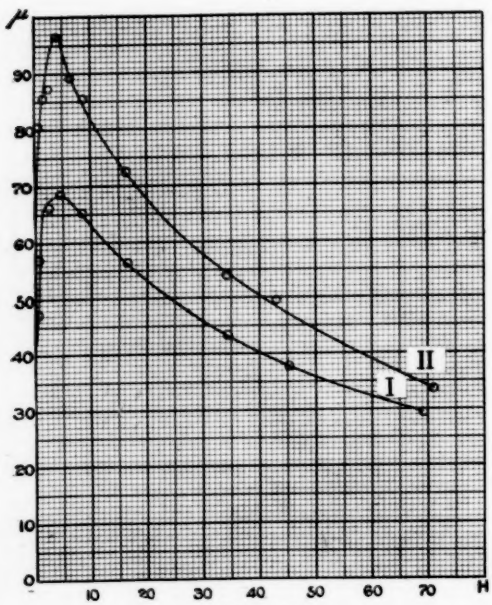


Fig. 17.

Examples of these measurements with alloys No. III*a* and No. IV*a* and No. IV*b*, are given in Table XII.

The values of the permeability of alloy No. III. were obtained when the specimen had become cool after being heated to 107° C. and kept at this temperature for 40 hours. A permeability curve repre-

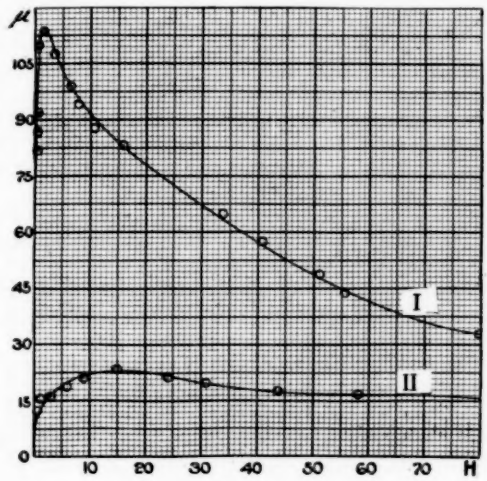


Fig. 18.

senting the results is given in Fig. 16, and a comparison with the curve made in the initial test which is plotted in the same figure shows that although the magnetic properties of the specimens were

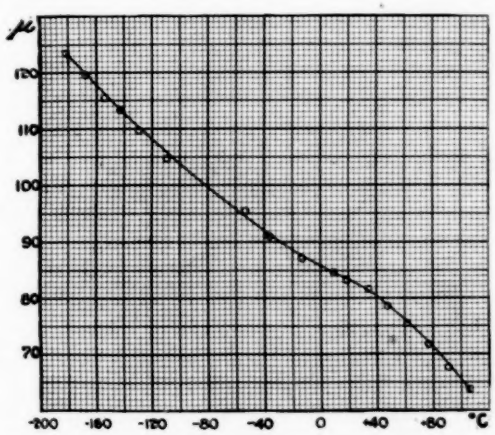


Fig. 19.

not very marked the heating nevertheless produced a very considerable increase in the permeability.

This increase in the permeability is also shown by the numbers for alloy No. IV*a*. Curve I., Fig. 17, shows the permeability of the alloy in different fields when freshly cast, and curve II. its permeability in the same fields after it had been repeatedly heated up to 100° C. and cooled to the temperature of liquid air.

The results obtained with alloy No. IV*b* show the effect of raising the alloy to a red heat and then allowing it to cool slowly. This specimen in the initial tests showed the highest permeability of all the rings, but after being treated as described a very considerable decrease in the permeability ensued. Curves I. and II. represent respectively the initial and final tests with this specimen.

In the interval between which the results with alloy IV*a* recorded in Fig. 17 were obtained, this specimen was examined under a constant magnetizing force of 3.934 C.G.S. units for temperatures ranging from — 182° C. to 105° C. The temperatures were estimated from variations in the resistances of a platinum wire wound round the ring between the primary and secondary coils. The results of these measurements are given in Table XIII. and a curve representing them is given in Fig. 19.

TABLE XIII.

Alloy No. IVa. H = 3.934 C.G.S. units.

Temperature.	B	μ	Temperature.	B	μ
−182	487	123.7	0	338.5	86
−169	471.8	119.9	8.2	334.2	84.9
−154	454.3	115.4	19.6	327.6	83.2
−143.5	445.6	113.2	35.3	321.1	81.6
−131	432.5	109.9	48	308.8	78.5
−110.5	410.6	104.3	63.1	297	75.5
−53	375.7	95.4	69	288.3	73.2
−46	364.8	92.7	76.4	281.8	71.6
−36	358.2	91	91.6	266.5	67.7
−13.5	342.9	87.1	105	521.21	63.8

The permeability, it will be seen, showed a steady increase as the temperature was lowered.

The results of these various tests, it will be seen are in accordance with the observations of other investigators, who have found that the permeability of these alloys can be considerably increased by continued heating at moderate temperatures.

The results obtained with alloy No. IV*a* lends support to the view expressed by Hill, that when these alloys are raised to a temperature higher than the transformation point, and then cooled, they possess a permeability which is largely determined by the temper ature from which the cooling took place. This alloy when cooled from the melting point as curve I., Fig. 18, shows, possessed a permeability very much higher than that which it had when cooled from a red heat.

This suggests a parallel between the behavior of these alloys and those studied by Heycock and Neville.[1] In their investigations it was found that the structure of an alloy at any temperature could be ascertained very closely by suddenly chilling it from that temperature. It is possible that these alloys under varying conditions of temperature, may pass through different phases, some of which may be magnetic, and others not. From the general results of the present investigations it is evident that the magnetic properties of the different alloys are intimately associated with their molecular structure, and since the structure of an alloy at any temperature can be ascertained by rapid cooling from that temperature, it is possible that the magnetic properties of the alloy may also be investigated by the same procedure. The results obtained by Hill and other investigators would seem to point in that direction.

PHYSICAL LABORATORY,
UNIVERSITY OF TORONTO, December 20, 1906.

[1] Rapports Congrès Int. de Phys., Paris, 1900, p. 131.

SOME NEWLY OBSERVED MANIFESTATIONS OF FORCES IN THE INTERIOR OF AN ELECTRIC CONDUCTOR.[1]

BY EDWIN F. NORTHRUP.

SOME months ago, my friend, Carl Hering, described to me a surprising and apparently new phenomenon which he had observed. He found, in passing a relatively large alternating current through a non-electrolytic, liquid conductor contained in a trough, that the liquid contracted in cross-section and flowed up hill lengthwise of the trough, climbing up upon the electrodes. With a further increase of current, he found that this contraction of cross-section became so great at one point that a deep depression was formed in the liquid with steeply-inclined sides like the letter V. This depression extended in the case of a liquid metal as deep as six inches. With a current of constant value, the condition was a stable one, but the liquid on the inclined surfaces showed great agitation. With a still greater increase of current, the depression reached the bottom of the trough, thereby rupturing the circuit; this, of course, resulted in the liquid flowing together again, and again breaking, a violent interrupter being thus formed. He thus concluded that there is a limit to the current which it is possible to pass through a liquid conductor and that this limit may be reached before the volitalization limit. He suggested that the blowing of fuse wires and the action of the Caldwell interrupter may, perhaps, be aided by this phenomenon. Mr. Hering suggested the idea that this contraction was probably due to the elastic action of the lines of magnetic force which encircle the conductor, which lines, he said, acted on the conductor like stretched rubber bands, tending to compress it, especially at its weakest point. As the action of the forces on the conductor is to squeeze or pinch it, he jocosely called it the "pinch phenomenon."

The phenomenon described appeared to the writer to have a considerable theoretical interest. The theory and the experiments de-

[1] A paper presented at the New York meeting of the Physical Society, March 2, 1907.

scribed in the following are the results of the writer's analysis of the problem. In the development of the theory which follows, he has been much assisted by the discussions which he had with Mr. Hering who offered many useful suggestions.

Depression Obtained with a Small Current.

To show the phenomenon observed by Mr. Hering, the writer devised several laboratory experiments in which mercury was used as the liquid conductor. In a thick sheet of hard rubber, about five inches square, two square holes or depressions were cut. These depressions were about three quarters of an inch deep and about one and three quarter inches on a side. Their two adjacent sides were parallel and about one half inch apart. A channel one quarter inch wide and three quarter inch deep was cut so as to connect the depressions. The depressions and channel were nearly filled with mercury. Electrodes of brass placed in the depressions enabled a storage battery current to be passed in the mercury from one depression to the other through the narrow channel.

It was found that about 800 amperes caused the mercury in the channel to depress lengthwise of the channel in the form of a V, about one half inch below the general level of the mercury. A small increase in the current caused the angle of the depression to reach the bottom of the channel and rupture the circuit. The liquid would again flow together and again rupture thus forming a slow and irregular interrupter.

The following experiment was then devised by which the forces at work could be shown in a more striking manner and with much smaller currents.

The apparatus shown in Fig. 1 was constructed. This is a box made of wood. The box is divided as shown into two rectangular compartments which are connected by a deep channel. One side of the channel was formed of a sheet of transparent mica, so the height of any liquid in the box could be observed from the side. In each end of the box were fastened brass electrodes.

The interior of this box, thus arranged, was filled to a depth of two inches with a liquid alloy of potassium and sodium. The remaining space was filled with kerosene oil. The liquid alloy

used has a good electrical conductivity and a specific gravity but little greater than kerosene. Hence, if, when current is passed from one electrode to the other, in either direction, forces act to cause the liquid metal conductor to depress in the narrow channel, C, a

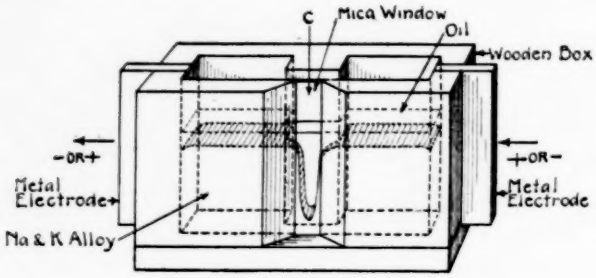


Fig. 1.

great depression should be obtained with a feeble force. It is evident that, while the conductor may depress, the level of the oil would remain unchanged. The experiment fully realized the results anticipated. Less than 180 amperes, when passed in either direction from one electrode to the other, caused the V-shaped depression to extend at least one and three quarter inches down. Regulating the current with a carbon rheostat enabled the depression to be held at any desired vertical depth. This proved that with a fixed current the depression formed in the liquid is stable. Fig. 1, drawn in perspective, shows the general appearance of what was observed.

PHYSICAL VIEWS OF THE FORCES IN AN ELECTRIC CONDUCTOR.

Several other experiments were made, but before describing them and explaining those just given, it will be useful to carefully consider the nature and magnitude of the forces which exist in the interior of a conductor carrying a steady electric current.

When a conductor carries an electric current, we may picture, with Faraday, a medium filled with lines of magnetic induction and consider all the forces connected therewith, or, we may consider, with Ampère, the conductor only, or the imaginary filaments of which it is composed, as giving rise to forces, which act through space according to laws ascertained by experiment. These two ways of viewing the matter are totally unlike, but, if properly treated,

both should lead to identical conclusions regarding the action of conductors and elements of conductors upon each other. We shall consider the forces, in which we are interested, in each of these two ways. The results obtained should check each other.

First conceive a very long straight conductor of small cross-section and very far separated from its return conductor. The action of such a conductor carrying a unidirectional current upon a small magnet in its neighborhood shows that the conductor is surrounded by lines of magnetic force. These lines are concentric circles with the conductor at their common center, and they pass normally through any plane passing through the conductor. The law of the diminution of the intensity of the field of force with increasing distance from the axis of the conductor is given by the following experiment, taken from Maxwell, § 478.

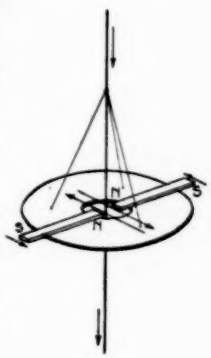


Fig. 2.

Two equal bar magnets are supported free to revolve around the conductor in the manner shown in Fig. 2. Under these conditions it is found that no movement of the magnets takes place whatever the strength of the current in the conductor. Hence, all the couples involved must balance. Thus, for the couples due to either magnet, we have

$$m_1 r_1 T_1 + m_2 r_2 T_2 = 0.$$

Here r_1 is the axial distance to the pole m_1 and r_2 to the pole m_2. T_1 and T_2 are the field intensities at these points. Other experiments show that in any magnet,

$$m_1 + m_2 = 0.$$

Hence

$$\frac{T_1}{T_2} = \frac{r_2}{r_1}.$$

Or the intensity of the field about an infinitely long straight conductor is inversely as the distance from the axis of the conductor.

Experiment also shows that the intensity of the field at any point in the neighborhood and outside of a conductor depends upon the

strength of the current. This intensity may be taken as the measure of the current and in the electro-magnetic system of measurement,

$$T = \frac{2I}{r}, \tag{1}$$

where I is the strength of the current in the conductor, and r is the distance from the axis of the conductor to any point outside of the conductor where the intensity of the field is measured.

When the current in the conductor disappears, the lines of force which encircle it disappear, leading to the conclusion that to have lines of magnetic force a current must exist which they can encircle. The above experimental facts suffice for determining the forces acting both in and around a non-magnetic conductor carrying a steady unidirectional current.

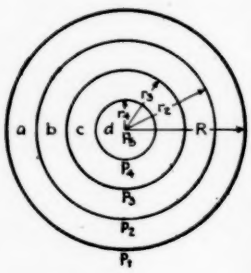


Fig. 3.

Besides the field of force exterior to the conductor, there is a field of force in the substance of the conductor. The intensity of this field, at any distance from the axis of a conductor of circular cross-section and of non-magnetic material, is easily found as follows: Let the outer circle of Fig. 3 represent the cross-section of such a conductor. Conceive this conductor to consist of concentric shells of internal radii, as r_2, r_3, r_4. Then at point p_1, located on the surface of the conductor, the intensity of the field will be,

$$T_1 = \frac{2I}{R},$$

I being the current in the entire conductor, and R its radius. At a point p_2 in the interior of the conductor, no line of force or field intensity can exist which is contributed by the outer shell, a. For we conceive p_2 to lie on the inside surface of the shell a, and on the interior of a hollow-conducting cylinder no lines of force can exist due to the current in the cylinder. This is proved by the experimental fact that a small magnet suspended in the interior of such a cylinder is uninfluenced by current in the cylinder. Moreover, lines of force cannot exist, because in the hollow space of the cylinder

there are no currents which they can surround. Every line of magnetic force, due to a current, must surround current. It is concluded, therefore, that any field which exists at p_2 must be due to the current in that portion of the conductor having a radius r_2. Call i_2 the total current in that portion of the conductor which has a radius r_2 and we have the intensity of the field at p_2,

$$T_2 = \frac{2i_2}{r_2}.$$

Similarly, the intensities of the field at p_3 and p_4 are

$$T_3 = \frac{2i_3}{r_3}, \quad T_4 = \frac{2i_4}{r_4},$$

or, in general,

$$T_r = \frac{2i_r}{r}.$$

If it be assumed that the distribution of the current throughout the cross-section of the conductor is uniform and we make

$I = \pi R^2$, we will have

$i_2 = \pi r_2^2$, or, in general, $i_r = \pi r^2$,

Then it follows that

$$i_r = \frac{r^2}{R^2} I = \frac{rT_r}{2}$$

where T_r is the intensity at distance r from the axis due to the current i_r in the circle of this radius. Hence we obtain

$$T_r = \frac{2Ir}{R^2}. \tag{2}$$

By similar reasoning, it can be shown that the intensity of the field in the metal at any point p, distant r from the axis of a hollow cylinder of internal radius d and external radius R is

$$T = 2I \frac{r^2 - d^2}{(R^2 - d^2)r}. \tag{3}$$

If $d = 0$, this expression becomes the same as the one above for a solid conductor.

EXPERIMENT TO SHOW THE ACTION OF THE FIELD IN THE INTERIOR OF A CONDUCTOR.

Since in the interior of a solid cylindrical conductor the field intensity *increases* from zero at the axis to a maximum at the circumference, a condition exists the reverse of that on the outside of the conductor where the intensity of the field *decreases* with the distance from the axis. To show this difference the following experiment was arranged.

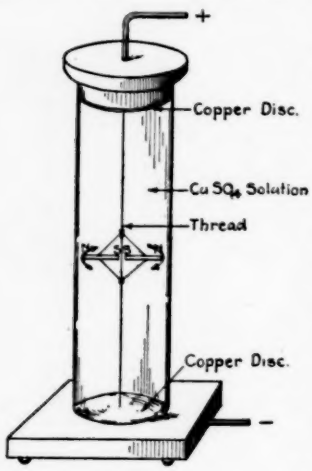


Fig. 4.

A glass tube 14 inches long and three inches inside diameter was fastened in a vertical position on a wooden base. In the top and bottom ends of the tube, copper disks were fitted. To each of these a lead wire was fastened, one passing out of the top and one out of the bottom of the tube. Between the top and bottom disks a silk thread was tightly stretched and located on the axis of the tube. At the center of the cylinder two small magnets were placed with their south poles pointing to the axis and their north poles to the circumference.

The magnets were fastened together by a light wire frame and supported by the thread in such a manner that they might rotate freely in a horizontal plane about the vertical thread as an axis. The arrangement described is made clear by referring to the diagram, Fig. 4. The glass cylinder was filled quite full with a strong solution of copper sulphate. A moderate current could be passed through the solution between the copper disks without generating gas.

As was expected, the magnets began to rotate as soon as current was sent through the liquid. When the current passed from the top to the bottom, the rotation was clockwise. Reversing the current reversed the rotation. With five amperes of current, the magnets made about two complete revolutions a minute. This result is demanded by the distribution of the field intensity in the interior of a conductor. For, call m and $-m$ the pole strengths of one of the magnets, T_1 the intensity of the field where m is located

at the distance r_1 from the axis, and T_2 the intensity of the field where $-m$ is located at the distance r_2 from the axis. Then the couple acting to turn one of the magnets in a horizontal plane about the axis of the conductor is,

$$P = mr_1T_1 - mr_2T_2.$$

We have found

$$T_1 = \frac{2Ir_1}{R^2} \quad \text{and} \quad T_2 = \frac{2Ir_2}{R^2}$$

R being the radius of the conductor. Hence,

$$P = 2Im\frac{r_1^2 - r_2^2}{R^2},$$

is the couple acting upon one magnet, while twice this couple acts to turn the two magnets, if these are alike and similarly located in respect to the axis.

Physical View of the Pressure in the Interior of a Conductor.

Let Fig. 5 be a cross-sectional view of the liquid column of Fig. 4. Let f represent the section of a small wire, which extends throughout the length of the liquid column and parallel to its axis.

If the liquid column has a radius R and carries a current I, the wire, if located at a distance r from the axis of the liquid conductor, will be in a field of magnetic force of intensity

$$T_r = \frac{2Ir}{R^2}.$$

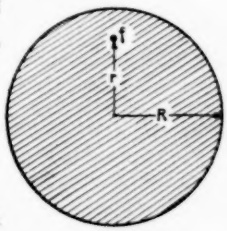


Fig. 5.

If current is passing *in the same direction* through the liquid conductor and the wire f, the wire will experience a force which will tend to move it across the circular lines of force in a direction to bring it toward and parallel to the axis of the liquid conductor. If the wire is free to move and the current density in it is greater than the current density in the liquid conductor, it will so move until it coincides with the axis of the liquid column.

This was proved experimentally as follows: In the apparatus shown in Fig. 4, a fine manganin wire of high resistance was placed in the liquid column. One end of the wire was fastened to the upper copper disk near its circumference and the other end to the corresponding point of the lower copper disk. The wire was slightly longer than the distance between the disks and hence hung loose between them. It was arranged to have it hang near the circumference of the liquid column. When current passed through the liquid column a current in the same direction and of greater current density also passed through the wire. On making the circuit, the loose wire was instantly urged from the circumference and formed a bow which bent toward the axis of the liquid column.

In place, now, of an actual wire in the inside of the liquid column, we may picture this liquid column as being made up of a great many filaments parallel to the column. Each filament will be a conductor which carries a current proportional to the total current and to its own cross-section. All the imaginary conducting filaments, except the one coinciding with the axis, will be in a field of magnetic force, the intensity of the field in which any filament lies being proportional to its distance from the axis. All the filaments, except the one on the axis, will experience a force which, if free to act, will urge them toward the axis of the conducting liquid column.

From this conception, it is evident that, if the conducting liquid column have elastic walls, it will contract so as to diminish in cross-section and in consequence increase in length. If the column cannot increase in length, it will tend to contract. In other words, there will be a pressure at the center greater than the pressure near the surface. This pressure is hydrostatic. Its existence and magnitude are shown by the following experiment.

EXPERIMENT TO SHOW PRESSURE IN THE INTERIOR OF A CONDUCTOR.

An apparatus was constructed which is shown in section in Fig. 6. T is the tube of fiber, and t another tube of fiber having an internal diameter of 2.54 cm; B and D are brass plugs tightly fitted in the ends of the inside tube. Heavy copper terminals enable these plugs to be joined to a battery of large current capacity. At the

center of the tube *t* was placed a fiber ring to reduce the cross-section of the tube at one place. The hole in the fiber ring had a diameter *2R* or a radius of .635 cm.

Through the sides of the tube and fiber ring several small holes, *h*, were drilled radially to the axis. The bottom of the annular space between the tubes *T* and *t* was closed by a fiber piece *P*. The two tubes were then filled with mercury to the level shown in the cut. According to the conception given above, when current is passing through the mercury column in the tube *t*, a hydrostatic pressure should exist in its interior, which is greatest along its axis. Hence, as the mobile conductor has no other escape than the hole *H* provided in the plug *D*, it should rise in this hole from the pressure at the axis, more being at the same time drawn in at the holes, *h*. Sufficient pressure should raise the mercury in the hole *H* until it overflows the upper surface of the plug *D*, and falls back into the well from whence it came. Thus, a continuous stream of mercury should flow by the path, through the holes *h*, up the tube *t* and the hole *H*, into the well and thence again to the holes *h*.

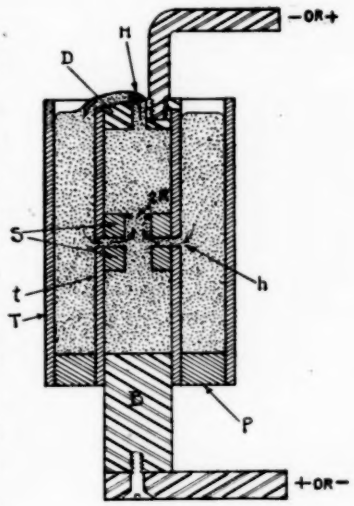


Fig. 6.

In trying this experiment, these results were amply realized. With about 1,800 amperes, the mercury flowed in a continuous and rapid stream, as described.

To test the magnitude of the pressure, a glass tube was tightly fitted in the hole *H*, and the mercury column was found to stand, with 1,800 amperes, six tenths of an inch in the tube above the general level of the mercury in the well.

So large a current soon heated the mercury and it was not found practical to keep it on more than a minute or two at a time. It is scarcely necessary to remark that the flow and the rise of the mercury is independent of the direction of the current.

It was thought that iron might be used to increase the induction

and so get the mercury to flow continuously with less current. The previous experiment was modified as follows.

A disk of soft iron was inserted in the narrow portion of the tube *t* in the manner shown in Fig. 7, *a* being a vertical section and *b* a horizontal section of the middle portion of the tube *t* and its fittings. A slot was cut in the iron disk parallel to its axis. This slot was 1.6 mm. wide and extended in radically to the center of the disk. The iron disk was located in the narrow portion of the tube *t* so that the end of the slot registered with one of the holes, *h*, Fig. 6. It is evident that the intensity of the magnetic field in the space of this narrow slot should be greatly increased by the presence of the iron. Also that the conducting mercury filaments in the slot should be urged with increased force toward the axis with a resulting greater hydrostatic pressure at the axis.

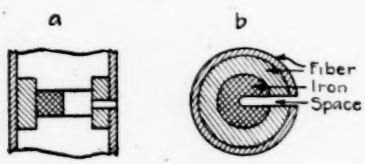


Fig. 7.

In trying the experiment, it was found that the force previously obtained was about doubled and that with 1,800 amperes the mercury would rise in the glass tube about one and a quarter inches. With 900 amperes and with the tube removed, the mercury would flow continuously, the heating being now reduced so that the current could be kept on for many minutes.

LAW OF THE PRESSURE IN THE INTERIOR OF A CONDUCTOR.

Let Fig. 8 represent the cross-section of a cylindrical conductor. Conceive this cross-section to be made up of a large number of annular spaces as 1, 2, 3, etc. Let the radial depth of each of these annular spaces be called dr. Let R be the radius of the cylindrical conductor, and r the radial distance from the axis to any point within the circular cross-section. It should further be supposed that the conductor is of very great length, having its axis perpendicular to the plane of the paper, and far removed from any return circuit. By the theory and experiments cited above, all portions of the conductor, as the annular sections, 1, 2, 3, etc., are under the influence of a force, when the conductor is carrying a current, which

tends to urge them radially toward the axis. This force, as measured in dynes, will have at any point distant r from the axis, a definite value per unit of area. The total force or pressure which acts upon the surface of any imaginary cylinder of radius r and length l, will be, evidently, the force per unit of area multiplied by the total area of a length l, of this imaginary cylinder.

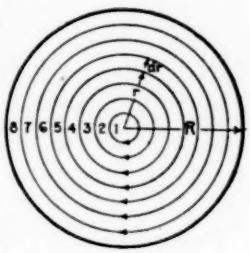


Fig. 8.

It is first required, to find the force, g, per unit of area, at any point within the conductor distant r from the axis, and second, the force or pressure P on the surface of an imaginary cylinder of radius r, and axial length l.

This problem may be solved by conceiving the action of the magnetic field on all portions of the conductor carrying current, or by considering only the mutual attractions of all the elements of the conductor.

By the first method, we may proceed as follows:

At a point within the conductor distant r from the axis the intensity of the magnetic field is by equation (2).

$$T_r = \frac{2Ir}{R^2},$$

where I is the total current flowing in the conductor. The lines of force of this field are circles, having as their common axis, the axis of the conductor.

Consider any single annular space as 5. It will carry current di, which is in the same ratio to the total current as the area of the annular space is to the total area of the cross-section of the conductor. If we call da the area of the annular space which has an inside boundary of radius r, we have

$$da = \pi(r + dr)^2 - \pi r^2 = 2\pi r dr,$$

since the square of dr can be neglected. Then as πR^2 is the total area, we obtain

$$di = \frac{2Irdr}{R^2}.$$

This current, di, is disposed in a field of intensity

$$T_r = \frac{2Ir}{R^2}.$$

If the current I flows downward, the lines of force would act on a unit north pole to move it in a clockwise direction around the axis; and an element of the conductor carrying a downward current would tend to move radially toward the axis. If the direction of the current were reversed, the direction of the magnetic lines would be reversed but the current elements would still tend to move toward the axis. The force in dynes with which a length l of any conductor is acted upon to move at right angles to the lines of magnetic force is, in electro-magnetic measure, numerically equal to the product of the length of the conductor, to the strength of the current in the conductor and to the field intensity where the conductor is located.[1] Thus calling dF the force with which the element carrying the current di tends to move radially toward the axis, we have

$$dF = ldiT_r = \frac{4I^2lr^2dr}{R^4}.$$

It should be noted that the force, dF, is distributed so as to act normally on a surface the area of which is $2\pi rl$. Hence, calling dg the force per unit of area, or the force intensity acting radially inward, we have

$$\frac{dF}{2\pi rl} = dg = \frac{2I^2rdr}{\pi R^4}.$$

This is the intensity of the pressure at distance r from the axis, due to the current in a single annular space of radial depth, dr. It is necessary in order to obtain the total intensity, g, at the distance r from the axis, to take the sum of the force intensities due to the currents in all the annular spaces, each of radial depth dr, which lie in the space included between the radius R and the radius r. This sum is given by the integral

$$g = \frac{2I^2}{\pi R^4}\int_r^R rdr.$$

[1] Consult Maxwell, Vol. II., § 490.

Performing the integration, we have

$$g = \frac{I^2}{\pi R^4}(R^2 - r^2). \tag{5}$$

Equation (5) gives the inward radial pressure per unit of area on the material of the conductor itself at any distance r from the axis of the conductor.

If I is the current per unit of cross-section, or the current density in the conductor, $I^2 = I_1^2 \pi^2 R^4$, and we obtain as another expression for the pressure

$$g = \pi I_1^2 (R^2 - r^2). \tag{6}$$

The pressure at the center, where $r = 0$, is therefore equal to the area of the cross-section of the conductor multiplied by the square of the current density.

The total pressure on the surface of an imaginary cylinder of length, l, and radius, r is, evidently, the force per unit of area multiplied by the total area of the surface of the cylinder considered. Calling P this pressure, we have:

$$P = \frac{2lI^2 r}{R^4}(R^2 - r^2) \tag{7}$$

or

$$P = 2\pi l I_1^2 r(R^2 - r^2). \tag{8}$$

These same expressions are easily obtained by conceiving the imaginary conducting shells to exert an attraction according to the law of mutual attraction between currents flowing parallel and in the same direction. The law of attraction between any two infinitely long parallel conductors distant r from each other and carrying currents i and i_1 is

$$F = 2ii_1 \frac{l}{r}. \tag{9}$$

Here l is the portion considered of the length of *one* of the conductors.[1]

In the case under consideration, we can take one of the conductors to be the cylinder of radius r, and the other conductor to be the annular area of inside radius r and outside radius $r + dr$.

[1] See Maxwell, Vol. II., § 495.

If I is the total current in the large conductor of radius R, the current carried by the conductor of radius r, is

$$i = \frac{r^2}{R^2} I,$$

uniform current density being supposed.

$2\pi r dr$ is, as previously found, the cross-section of the annular area and the current which it carries is therefore

$$di = \frac{2rdr}{R^2} I.$$

The currents i and di attract and produce a pressure over the surface of the cylinder of radius r. Calling dF this pressure, we have

$$dF = \frac{2idil}{r} = \frac{4lI^2r^2dr}{R^4}.$$

The pressure per unit of area is:

$$\frac{dF}{2\pi rl} = dg = \frac{2I^2rdr}{\pi R^4},$$

whence, in integrating, we obtain

$$g = \frac{2I^2}{\pi R^4}\int_r^R rdr = \frac{I^2}{\pi R^4}(R^2 - r^2)$$

and

$$P = \frac{2lI^2r}{R^4}(R^2 - r^2).$$

These last two expressions are seen to be identical with (5) and (7).

ATTRACTION IN THE INTERIOR OF A CONDUCTOR.

The attraction in the interior of a cylindrical conductor carrying I_1 units of current per unit of cross-section which would be exerted upon a unit length of a unit of current distant r from the axis, may be found as follows:

In equation (9) let $l = 1$, and we have

$$F = \frac{2ii_1}{r}.$$

The current carried by the conductor of radius r is

$$i = \frac{r^2}{R^2} I,$$

which combined with the above gives

$$F = \frac{2ri_1 I}{R^2}.$$

Calling I_1 the current per unit of cross-section of the conductor and taking i_1 equal to unity, we find

$$F_t = 2\pi I_1 r. \qquad (10)$$

Equation (10) expresses the attraction in the interior of a conductor carrying current. It is analogous to the gravitational attraction in the interior of a body of the same form as the conductor, due to its mass only.

GRAVITATIONAL ANALOGY.

It is interesting to compare the forces produced by an electric current in a conductor of circular cross-section with the forces produced by gravity in the interior of an infinitely long cylinder of the same radius and section as the electric conductor.

For the sake of comparing methods, we may use Poisson's equation to find the expressions giving the forces in the cylinder produced by gravity.

It is shown in Thomson and Tait's Natural Philosophy, Vol. II., page 37, that for the case considered, this equation takes the form

$$rF = C - 4\pi M \int_r^R r dr = C - 2\pi M(R^2 - r^2).$$

Here F is the force of gravitational attraction which acts radially toward the axis of the cylinder; M is the mass per unit of volume, and C a constant. Since we know, by symmetry, that F is zero at the axis, where r is 0, we obtain

$$C = 2\pi MR^2.$$

Hence

$$F = 2\pi Mr. \qquad (11)$$

Equation (11) is an expression of the same form as (10).

If we express F in dynes and r in centimeters, we must take for the unit of mass, that mass which at one centimeter from a like mass will cause an attraction between the two of one dyne. The value of this unit in grams is given in Everett's "C.G.S. System of Units," page 73, 1891 edition, as equal to 3,928. Hence, if m be a mass expressed in grams, we have

$$M = \frac{m}{3928},$$

or

$$F = \frac{2\pi mr}{3928} \text{ dynes.} \tag{12}$$

We shall now have for the force acting on the matter in an annular space of radius r, radial depth dr and length l,

$$dF_1 = \frac{2\pi mr}{3928} \cdot \frac{2\pi lmrdr}{3928} = \frac{4\pi^2 lm^2 r^2 dr}{3928^2}.$$

The intensity of this force, or the dynes pressure per unit of area at distance r from the axis is

$$\frac{dF_1}{2\pi rl} = dg_1 = \frac{2\pi m^2 rdr}{3928^2}.$$

And by integrating, we obtain

$$g_1 = \frac{2\pi m^2}{3928^2}\int_r^R rdr = \frac{\pi m^2}{3928^2}(R^2 - r^2). \tag{13}$$

Or, if we call G the number of grams of matter in a unit of length of the cylinder of radius R,

$$g_1 = \frac{G^2}{\pi R^4\, 3928^2}(R^2 - r^2). \tag{14}$$

Comparing equations (5) and (14), we note that

$$g = 1.513 \times 10^7 g_1 \frac{I^2}{G^2}, \tag{15}$$

which means that the hydrostatic pressure in the interior of a conductor at any distance r from the axis when carrying I electromag-

netic units of current is 1.513×10^7 times the pressure in this conductor, having G grams of matter per unit length, due to gravitational attraction alone.

If we compare equations (10) and (12), we note that the attraction due to the electric current, which would act on a unit length of a unit current, is of the same nature and in the same direction as the gravitational attraction which would act on a gram of matter, but 3,928 times as great, the number of units of current per unit of cross-section of the conductor being equal to the number of grams of matter per unit of cross-section in a unit of length of the cylinder.

Graphs.

The leading features respecting the forces discussed above are summarized in the curves shown in Fig. 9. The curves in full line apply to the cylindrical conductor which is shown in cross-section in full line. The curves in broken line, and the curve 2 apply to the conductor shown in broken line, having one half the diameter of the first conductor. In both cases, the larger and smaller conductor are supposed to carry the *same* current. Curve 1 gives the intensity of the field in the interior of the conductor. Curve 2, the intensity of the field exterior to the conductor. Curve 5, expresses the pressure in dynes per square centimeter. This pressure is a maximum at the center and zero at the circumference. At one quarter of the distance from the center to the circumference, the pressure has diminished but 6¼ per cent. — a fact having importance in certain practical constructions.

When the conductor is diminished in diameter to one half, its area is one fourth and the current density is increased four times, hence it follows from equation (6) that the pressure at the center is increased four times. This is shown by curve 6. The intensity of the field in the inside of the conductor is doubled, curve 4. On the outside of the conductor, the intensity of the field is not altered beyond the radius of the larger conductor, curve 2; but in the space outside the smaller and inside the larger, there is a field the intensity of which is given by curve 3.

From what has been given, the following deductions are easily made.

Suppose a conductor to have originally a radius R. Imagine the conductor to contract in cross-section until its radius becomes aR, a being a number less than unity. Further, assume the conductor

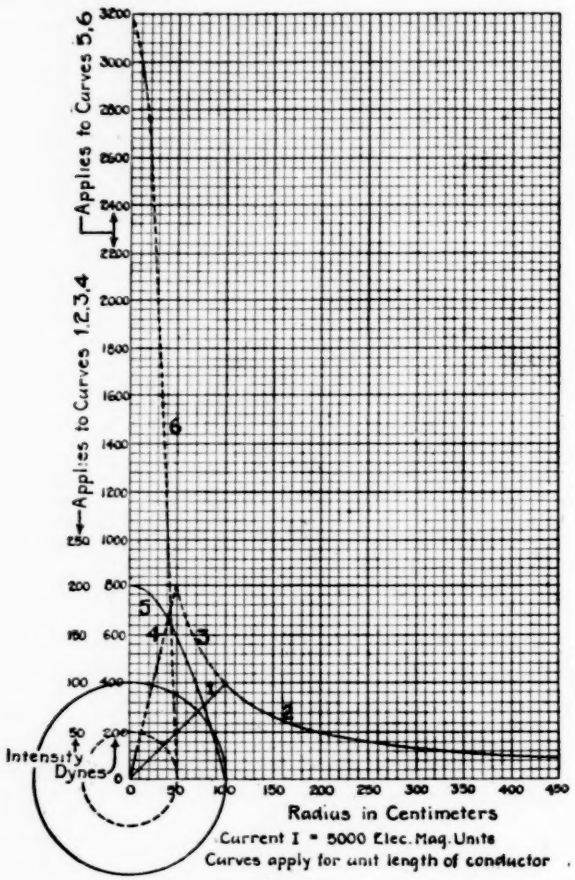

Applies to Curves 5,6
Applies to Curves 1,2,3,4
Intensity Dynes
Radius in Centimeters
Current I = 5000 Elec. Mag. Units
Curves apply for unit length of conductor

Fig. 9.

to always carry the *same* total current I and to preserve a circular cross-section. Let a portion of the length of the conductor be called l. Then the total number of lines of force in the interior of a length, l, of the conductor is given by the expression

$$N = \frac{2Il}{(aR)^2}\int_0^{aR} rdr = lI. \qquad (16)$$

Hence, we conclude that, however, the conductor may change in

diameter, the total number of lines of force in its interior remains the same, provided the current is always the same, and that per unit length of the conductor the total number of lines of force in the conductor is numerically equal to the current in electromagnetic measure.

An inspection of curves 2 and 3 shows that when the conductor contracts from a radius R to a radius aR, the current remaining the same, lines of force are developed. These all lie in the medium around the conductor and their number per length l of the conductor is given by the expression

$$N_0 = 2Il \int_{aR}^{R} \frac{dr}{r} = 2Il \log_e \frac{1}{a}. \tag{17}$$

From this, it follows that the total number of lines exterior to the conductor is increased when the conductor diminishes in diameter, and as the number inside the conductor remains the same, we conclude that the coefficient of self-induction of a small wire is greater than that of a large wire.

It may be further concluded that, if the current be maintained constant, when the conductor diminishes in diameter, electromagnetic energy is stored up in the medium around the conductor.

It may be shown that the electromagnetic energy in the interior of a length l of the conductor is given by the expression

$$E = \frac{lI^2}{R^4} \int_0^R r^3 dr = \frac{lI^2}{4}. \tag{18}$$

Thus the energy stored in the conductor, as well as the number of lines of force, is independent of the diameter of the conductor. When the conductor contracts from radius R to radius aR, the energy added to the medium is stored in the space occupied by the conductor previous to its contraction less the space occupied by the conductor after its contraction. The energy so added per length l of the conductor is given by the expression

$$E_0 = I^2 l \int_{aR}^{R} \frac{dr}{r} = lI^2 \log_e \frac{1}{a}. \tag{19}$$

The above expressions (16), (17), (18) and (19) are given without

proof, for they may be readily obtained from the facts stated above, and the energy relations given by Maxwell.

ADDING PRESSURES IN SERIES.

The writer soon perceived that to obtain forces of sufficient magnitude to be useful in practical applications means should be devised whereby the internal pressure at one point in a conductor might be added to that in another point and so on. This result may be

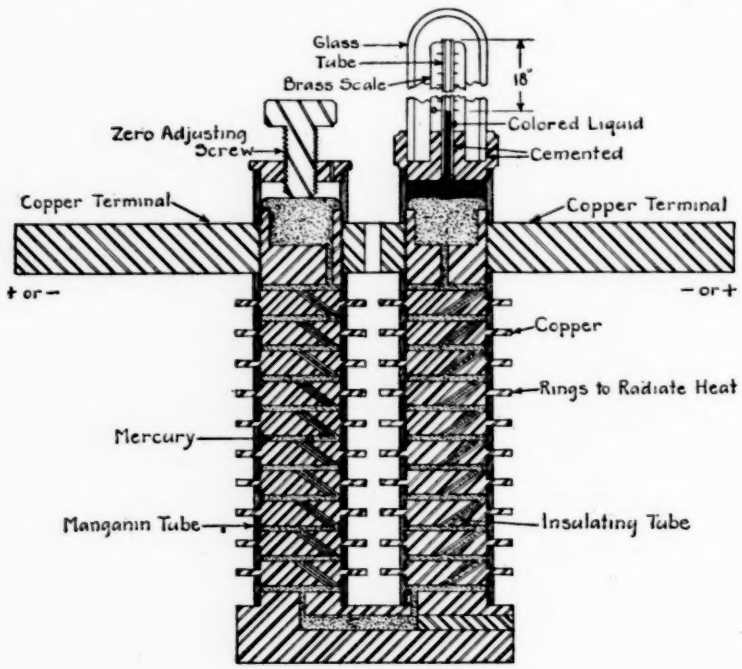
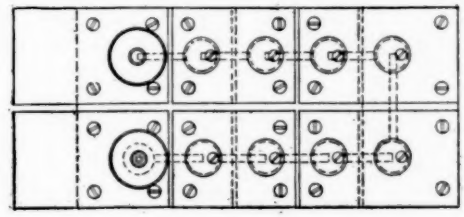
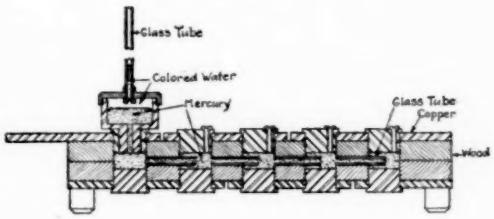


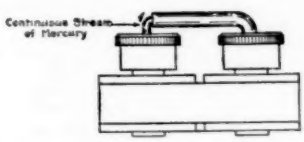
Fig. 10.

attained as follows: A cylindrical conductor is constructed which consists of alternate strata of solid and liquid, as copper and mercury. This heterogeneous conductor is tightly encased in a tube of insulating or high resistance material. In each of the solid portions of the conductor small tubes of insulating material are placed in a manner to connect the axis of one liquid layer with the circumference of the next liquid layer. This forms a continuous passage which goes in steps from the axis to the circumference of each liquid portion. A tube from one end of this conductor connects with a

reservoir of the liquid and another tube of glass is raised from the other end of the conductor which connects with the end liquid layer and in which the liquid may rise. Heavy copper terminals lead the current, which may be direct or alternating, in, and out of, the compound conductor. When the electric current flows, the pressures produced at the axes of the liquid portions of the conductor add in series and the liquid may be thus raised to useful heights in the glass tube. Fig. 10 shows in section such a contrivance which it is evident might serve as a direct or alternating current ammeter of large capacity. A construction similar to that described was made and operated, and is a useful alternating current ammeter.







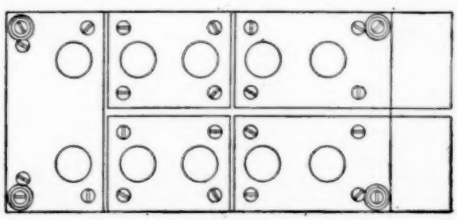


Fig. 11.

Fig. 11 shows another embodiment of the same principle. This apparatus was constructed and the calculated height of about 22 cm. to which the colored water would rise in the tube with 600 amperes was practically attained.

It should be noted that the hydrostatic pressure obtained is independent of the axial length of the liquid strata, while the heat developed by the current is directly as this length. With this consideration in mind, the axial lengths of the liquid portions of the apparatus shown in Fig. 11 were afterwards reduced to 1.6 millimeters each. The pressure was not lessened and the efficiency or cool running was greatly improved. The theoretical pressure or height of mercury that may be obtained in any particular case may be deduced easily from the curves (Fig. 9), or calculated from the expression

$$g_s = nAI_1^2 \tag{20}$$

which is derived from equation (6), when $r = 0$, $\pi R^2 = A$, the cross-section, $I_1 =$ the current density and $n =$ the number of times the pressures at the axis are added in series.

If C is the current density per square cm. expressed in amperes and g_p is the pressure in pounds per square cm. and A is the cross-section in square cm. the expression (20) takes the form

$$g_p = \frac{nAC^2}{44479100}. \tag{21}$$

In the apparatus shown in Fig. 11, $n = 10$, $A = 1.2$, $C^2 = 250{,}000$, which corresponds to nearly 600 amperes in the conductor. Whence we get $g_p = .067$ lb. per square cm. or .432 lb. per square inch. This pressure should give a rise of .88 inch of mercury or about 11.9 inches of water. The rise obtained with 600 amperes in the apparatus of Fig. 11 was about 9 inches, but this lessened height is due to the fact that the mercury itself had to raise some to produce the rise in the water. The point to note, however, is that with 600 amperes, the apparatus runs cool. Hence we can conclude that with a heterogeneous conductor of one half inch diameter and made up of 1,000 cells, or mercury sections of say, 1 mm. length each, we could obtain with 600 amperes the large pressure of 43.2 lbs. per square inch, and this without iron in the circuit. This pressure would sustain a column of mercury over seven feet high.

Some Final Remarks.

One is naturally led to inquire if the recognition of this force that exists in the interior of electric conductors and the establishment of its laws of action is likely to result in new and useful applications. It is quite certain that this force which depends only on the square of the current and linear dimensions may be used as an accurate measure of the current, either direct or alternating. As the forces can be enhanced by a proper disposition of iron in the heterogeneous circuit, motors without brushes or slip rings may be made operative. And remembering that power is the product of current and electromotive force, it is not impossible, even if large operative currents are required, that fair efficiencies may be obtained.

Further thought along an engineering line may bring about developments of a unique and useful character.

THE DETERMINATION OF POTENTIAL BY CARBON PENCILS.

BY C. D. CHILD.

AN investigation of the determination of the potential of hot gases by carbon pencils has shown that error may often be introduced when measurements are made by this means. Such errors appear to have been made by the present writer in the work on the magnetic deflection of the arc[1] and possibly by Stark, Retschinsky and Schoposchnikoff[2] in their work on the relative velocities of the ions of the carbon arc. The phenomena which give rise to these errors may be described as follows.

Potential Difference between hot Carbon and surrounding Bodies. It has already been shown by the present writer that when the temperature of carbon in air is gradually raised, it first discharges positive electricity but not negative, that secondly it discharges both positive and negative, but the positive in greater amount, and finally that it discharges negative more readily than positive In the similiar case with hot platinum wire it was shown by Elster and Geitel[3] that if there was no external E.M.F. between the hot wire and a neighboring insulated body, the wire was positive as compared with the other body, when it first became luminous and negative when it became white hot. It seemed probable that the same thing would occur with carbon.

To test this a carbon pencil, 1.6 mm. in diameter, was placed inside an insulated brass cylinder 2 cm. in diameter. The pencil was heated by passing a current through it. A high resistance shunt was placed about the pencil and the middle of this shunt was connected to one pair of the quadrants of an electrometer and the cylinder to the other pair. When the pencil first became luminous,

[1] PHYS. REV., 20, 100.
[2] Drude's Ann., 18, 230.
[3] Wied. Ann., 31, 109.

it was negative as compared with the cylinder, and when white hot, it was positive. It varied approximately from minus four to plus three volts. Thus it may happen that in some cases the hotter of two carbons will be positive as compared with the cooler, while in others it will be negative. When one is red hot while the other is non-luminous, the hotter will be negative. When one is white hot while the other is either cool or red hot, the hotter will be positive.

When potential is measured by hot carbon pencils, error may be introduced by overlooking either of two phenomena ; one, that at certain temperatures carbon discharges positive electricity the more easily, while at others the negative is more easily discharged ; the other, that there is an E.M.F. between carbons of different temperatures and that in some cases the hotter will be positive, while in others it will be negative.

Magnetic Deflection of the Arc. — Let us consider the bearing of these facts on the experiment of the magnetic deflection of the arc to which reference has been made. It was apparently shown in that article that when the arc was deflected to one side by a magnet, the side toward which it moved assumed a higher potential than the opposite one. In making the measurements the potential was determined by placing carbon pencils in the arc. It was evident that the pencil toward which the arc was deflected would become hotter than the other, and it was recognized at the time that this might be the cause of the phenomena. This was tested by blowing the arc to one side by a draught of air and the results thus obtained were opposite to those observed with deflection by a magnet. The conclusion seemed evident that the phenomena were not due merely to differences in temperature.

However, such a conclusion was probably incorrect. At that time it was found that when the arc was blown to one side by a draught of air and the potential difference between the two pencils measured by a voltmeter, errors were liable to be incurred because of the increased resistance between the two pencils when one was cool. An electrometer was, therefore, substituted for the voltmeter. This electrometer had a period of 14 sec. and in order to observe any effect it was necessary to keep the arc blown to one side for some time, and in fact, it was blown entirely to one side, in order

that there might be no question as to which carbon was the hotter. Apparently it happened that when the arc was deflected by the magnet both pencils remained luminous and that the one which was very hot was positive as compared with the cooler one, but that when the arc was deflected by draughts of air, the side from which the arc was blown was not luminous and the other was luminous but not white hot, so that the hotter pencil in this case was negative.

In order to test this further the experiment with the arc blown to one side by a draught of air was repeated, care being taken not to deflect the arc more than a slight amount, and it was found that the effect was then the same as when the arc was deflected by a magnet. For this purpose it was found best to use a millivoltmeter with a needle having a short period. With this the arc could be deflected and the effect observed before the pencil had become non-luminous.

In measuring the potential differences with a voltmeter the effects produced are largely due to changes in conductivity at the surfaces of the pencils, since the resistance of the voltmeter is often small as compared with that between the pencils. This, however, would give the same apparent changes in E.M.F. as the real changes described above. If two pencils are placed in the arc and connected by a voltmeter, it is impossible to place them so that for an appreciable length of time there will be no reading of the voltmeter. If they are placed as in Fig. 1, where *a* is nearer to the positive carbon, *a* will have a higher potential and a current will flow through the voltmeter from *a* to *b*. *a* is here receiving positive ions from the arc and giving negative ions to it, *b* is receiving negative ions and giving off positive ones. If now the arc were to be deflected slightly to the right *a* will be hotter and better able to give off negative ions, while *b* may not become so cold as to change appreciably its discharging power. The current through the voltmeter will increase, as if *a* had acquired a higher potential.

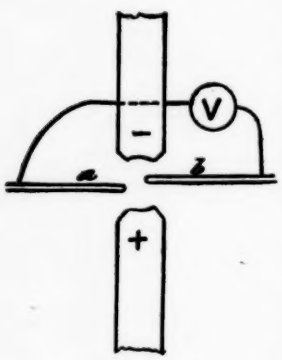


Fig. 1.

On the other hand, if the arc is deflected to the left, *a* will be less able to discharge negative ions and the voltmeter will read less than before, as if the potential of *a* had been lowered. So that whether we consider the effect which the temperature has on the potential of the pencils when we use an electrometer, or its effect on the conductivity near the pencil when a voltmeter is used, the apparent effect due to the change of temperature is the same.

Such an explanation as the preceding would account for the fact that the effect which had been observed with the magnetic deflection was less apparent with arcs between cored carbons, or with arcs into which salts were introduced, or with arcs in a vacuum. In each of these cases the arcs were not so hot and probably neither of the pencils reached the temperature where they gave off negative ions in abundance. In a vacuum the pencils may even be non-luminous while in the arc.

Measurement of Potential to Determine Relative Velocities of Positive and Negative Ions. — Passing now to the experiment of Stark, Retschinsky, and Schaposchnikoff, their method may be described as follows. If there is uniform ionization between the plates *A* and *B*, Fig. 2, and if a potential difference be maintained between them, the curve indicating the potential at various points will be similar to the curve *acb*. If the ionization is the same on both sides of the middle point *c*, and if the positive and negative ions have equal velocities, the drop in potential will be the same on the two sides. If, however, the negative ions have the greater velocity, other conditions being the same, the drop between *a* and *c* will be less than that between *c* and *b*. If the positive ions have the greater velocity, the drop between *a* and *c* will be the greater.

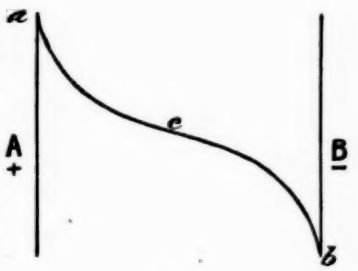


Fig. 2.

The writers to whom reference has been made attempted to find the relative velocities of the positive and negative ions by placing two small graphite pencils on opposite sides of the arc and between them a third pencil for testing the potential. All of these were

near enough to the arc, so that they became self-luminous. The two opposite pencils were connected to an E.M.F. of about 7 volts. By this means they found that the drop in potential at the negative pencil was the greater and they concluded that the negative ions moved the more rapidly.

It is possible, however, that a different conclusion should be drawn from these facts. If, as before, there were uniform ionization between A and B, Fig. 2, and the two kinds of ionization had the same velocities, and if in addition there was ionization at the surface of A, then the drop in potential at A would be less than than that at B. That is, the positive ions coming from A would neutralize the electrostatic effect of the negative ones coming toward A. Or to state it in other words, the conductivity near A being increased, the potential gradient there would be diminished. A similar statement may be made concerning the effect of ionization at B.

In the experiment which has been described there was ionization not only in the arc, but also at the surface of the graphite pencils. It might easily be complicated by these phenomena. This would be the more serious in this case, since a large part of the fall of potential between the graphite pencils occurs near the pencils. This, at least, is the condition which we find at the electrodes placed in a flame, especially when the electrodes are much cooler than the gas between them as they are with the arc. Under these conditions a slight increase in the conductivity near the electrodes will have a large effect on the drop in potential, and the drop may be the smaller at the electrode which gives off the greater number of ions quite irrespective of the velocity of the ions.

It is possible to test this matter, for if the results are due to the ionization at the surface and not to relative velocities of the ions, we should expect the drop in potential at the positive pencil to be greater than that at the negative, when the pencils are placed near the center of the arc. It should, however, be borne in mind that no matter how hot the point of the pencil may be, there will be some region further from the center of the arc, which will be just hot enough to give positive ions, but not negative, so that to have a preponderance of negative ions given off, it is necessary that a comparatively large part of the pencil should be very hot.

Method for Testing this Explanation. — The apparatus was arranged as shown in Fig. 3. The arc occurs between *A* and *B*, which were solid carbons 12 mm. in diameter. The carbon pencils *a* and *b* were inserted in or near the arc, the two being approximately equally distant from the center of the arc. These were 3 mm. in diameter. A potential difference was maintained between them by a battery *C*. *ef* is a resistance shunted between *a* and *b* and *d* was the middle point of this shunt. *c* is an exploring pencil placed in the center of the arc, and *E* is an electrometer for determining the potential difference between *d* and *c*. *D* is a reversing key. If the greater drop in potential is at the positive pencil, *c* will have a lower potential than *d*, whichever way the current will be flowing through the arc or between *a* and *b*.

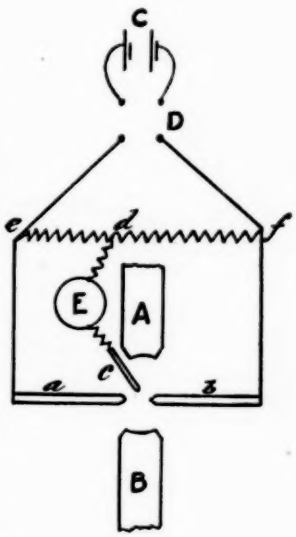


Fig. 3.

The experiment was first performed with a direct current through the arc of approximately 12 amperes, and a potential difference of 13 volts between *a* and *b*. The data taken are given in the following table, column 1 giving the distance between *a* and *b* in millimeters, column 2 the potential difference between *c* and *d* in volts, the sign indicating the potential of *c* as compared with *d*. The readings given in the table are the average of four readings, both the currents through the arc and through *a* and *b* being taken direct and reversed.

Distance Between Exploring Pencils, *a* and *b*.	Potential Difference Between *c* and *d*.
30	+ .4
25	+1.4
15	−1.9
10	−1.1
5	+1.0

In the first readings the exploring carbons were so far from the arc that they were non-luminous. In this case the result was not

complicated by ionization at the surface of the exploring carbon. This would indicate that the positive ions had a greater velocity than the negative. This was consistent with what had previously been observed, when studying the discharge from the carbon arc to a surrounding cylinder.[1] It was at that time found that with a constant potential difference between the arc and the surrounding cylinder the current to the cylinder was only one half as great when the cylinder was positive as when it was negative. Here with a constant current we find the drop at the anode to be greater than that at the cathode. Both of these phenomena would appear to indicate that the positive ions drawn from the arc move more rapidly than the negative.

When the distance between the exploring carbons was 15 mm., they were both luminous. The drop at the cathode was then the greater. This is in agreement with the results obtained by Stark, Retschinsky and Schoposchnikoff.

When the exploring carbons were 5 mm. apart the drop at the anode was again the greater. If these results depend only on the relative velocity of the positive and negative ions, they would indicate that at the center of the arc, the velocity of the positive ions is greater than that of the negative. It can hardly be believed that the velocity of the positive ions outside the arc is greater than that of the negative, that it is less in the cooler parts of the arc, and that it is again greater in the center. It seems more probable that any effect produced by the relative velocities of the ions is masked by the ionization at the surface of the carbon and that the phenomena observed is due to the relative ease with which the different ions are given off at different temperatures.

The experiments were then varied, first by changing the potential difference between *a* and *b* to 7 volts, secondly by using an alternating current for the arc, and third by using an alternating potential difference between *a* and *b*. In all of these cases the results were of the same general character as those given in the table.

When cored carbons were used the average potential at *c* was at all times above that of *d*, being more, however, when the distance between *a* and *b* was in the neighborhood of 15 mm. This is prob-

[1] Physical Review, 12, 143.

ably due to the fact that with cored carbons the temperature of the arc and of the exploring carbons is not as high as with solid carbons.

It should be noticed that if this explanation is correct, we are not at any time measuring the true drop in the potential at the different pencils, for the middle exploring pencil does not take the true potential of the hot vapor in which it is placed. It shows rather that the effects are too complicated to admit of an explanation depending merely on the relative velocities of the ions.

One might think at first that the difficulty here shown could be obviated to a great extent by taking greater potential differences between the exploring carbons, so that any potential difference between the middle exploring pencil and the gas would be negligible as compared with other potential differences. This, however, is impracticable, because an arc would start between the exploring pencils, if a large potential were maintained between them.

This experiment does not in any way throw doubt on the valuable work which Stark, Retschinsky, and Schoposchnikoff performed on the relative velocities of the ions in the mercury arc, for in that case the exploring pencils did not become luminous, nor does it *prove* that the negative ions have not the greater velocity in the carbon arc. It does, however, throw doubt on the certainty of their method of proof and leave it desirable that some further experiment be made on the subject.

Measurement of the Potential of the Arc Vapor. — The measurements which have been made on the potential of the vapor of the arc and on the drop in potential at the anode and at the cathode must also be subject to some error, because of this E.M.F. between the vapor and the exploring carbon pencil. However, it does not seem probable that such error is much, if any, greater than the errors which are know to exist. As far as is known to the present writer no attempt has been made to determine whether such error exists and no investigation has been made of the effect which may be produced on the apparent potential of the arc vapor when the exploring pencil is inserted for different distances into the arc.

Swinton's Experiment on the Relative Velocities of the Positive and Negative Ions. — It is also a question whether the experiment per-

formed by Swinton[1] proves anything concerning the relative velocity of the ions of the arc in air. He describes his experiment as follows: "One of the carbon electrodes was made hollow and was pierced axially by a small hole, and an insulated Faraday cylinder of brass, with its aperture in a line with and facing the aperture in the electrode, being fixed inside of the electrode. The insulated Faraday cylinder and the pierced electrode were connected together through a galvanometer, and it was found that when the opposite electrode was made negative, the pierced electrode being positive, a negative charge was imparted to the Faraday cylinder, while, if the opposite electrode was made positive and the pierced electrode was made negative, the Faraday cylinder acquired a positive charge."

He found that the negative charge was greater than the positive and from this concluded that the velocity of the negative ion was the greater. He would have found the same, however, if he had placed the exploring carbon in the arc, or any where else near the arc, instead of in the cylinder. Ions diffuse in all directions from the arc and since the potential difference between the vapor of the arc and the anode is greater than that between the vapor and the cathode, the potential difference between the exploring pencil and the anode would be greater than that between the pencil and the cathode, whenever it was placed in the same position with reference to the cathode as it had previously been placed with reference to the anode.

If penetrating anode and cathode rays were given off by the arc which could be studied, they would give indications concerning the velocities of the ions in the arc, but Swinton has himself shown[2] that there are at least no such rays which penetrate .00265 mm. of aluminum, and we have no evidence whatever that there are any which would penetrate even a millimeter of air. It seems, therefore, probable that the phenomena observed by Swinton was produced by the diffused ions, and that these indicated nothing concerning the relative velocities of the ions in the carbon arc at atmospheric pressure.

Summary. — We may summarize by saying that between hot

[1] Proc. Roy. Soc., A 76.
[2] Phil. Mag. (6), 11, 829.

carbon and surrounding bodies there is an E.M.F. which depends in amount and sign on the temperature of the carbon and that this E.M.F. is liable to introduce error in determining the potential of a hot gas.

Such an error was introduced by the present writer in an article on the arc in a magnetic field. In all probability the phenomena there described were due to such an E.M.F. It also seems probable that the phenomena observed by Stark, Retschinsky, and Schoposchnikoff in attempting to determine the relative velocity of the ions of the carbon arc in air were due to this cause rather than to differences in the velocities of the ions.

It also seems probable that the phenomena observed by Swinton were due to the diffusion of the ions in the arc and not to any anode or cathode rays. So that some definite determination of the relative velocity of ions in the carbon arc in air is a measurement yet to be made.

COLGATE UNIVERSITY,
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COLGATE UNIVERSITY,
February, 1907.

THE HIGH RESISTANCE CONTACT THERMO-ELECTRIC DETECTOR FOR ELECTRICAL WAVES.[1]

BY L. W. AUSTIN.

IF two metals standing far apart in the thermo-electric series be so brought in contact that their contact resistance is sufficiently high and if the surfaces of the metals are of such a nature that heat is not conducted away too rapidly from the point of contact, then the passage of electrical oscillations through the junction will produce direct electromotive forces which may be detected in a galvanometer or telephone in series with the thermo-element. The principle is extremely simple but in practice it is found that there are very few pairs of metals which give suitable contact conditions. If the contact is of too low resistance the heating is insufficient and if the contact is too loose as in the coherer the action is irregular, and the instrument less sensitive.

On account of its extreme position in the thermo-electric series tellurium seemed desirable for one of the metals and it was tried with a large number of others. Fair results were obtained with several of the more easily oxidized metals and also with carbon but with all these the action was irregular and adjustment difficult. Finally aluminium was fixed upon as being not only the most sensitive but by far the most reliable among the common metals. Silicon has also proved very satisfactory.

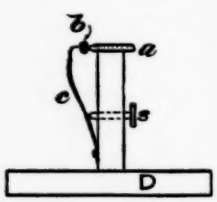

Fig. 1.

The instrument has been made in three forms. The first form is shown in Fig. 1. *D* is a hard rubber support at the top of the upright piece of which the pointed rod of aluminium *a* is mounted. *b* is a bead of tellurium melted onto the tip of a piece of brass wire the latter

[1] Briefly described in a letter to the Electrical World, Vol. XLVIII., p. 924, 1906.

A similar detector using silicon and copper was independently devised by G. W. Pickard, Electrical World, Vol. XLVIII., p. 1003, 1906.

being soldered to the top of the spring *c*. The closeness of the contact between *a* and *b* is regulated by the screw *s*. A second form was also made in which the aluminium is in the form of a rod fitted in bearings and rotated slowly by means of a pulley. The tellurium is pressed against it by a spring as before. The constantly changing contact point seems to keep the conditions more constant and does away with any necessity of readjustment even when heavy atmospheric discharges have passed through it. The sensitiveness is about the same in the two types. As certain portions of the tellurium surface seem to be more sensitive than others it is desirable in both forms to have the point of contact adjustable.[1]

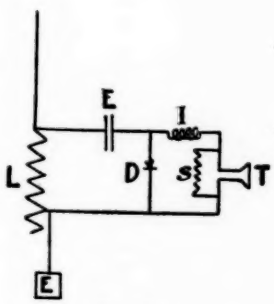

Fig. 2.

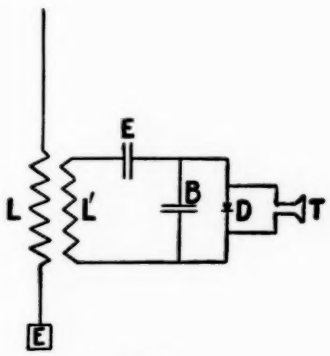

Fig. 3.

An exhaustive comparison of the thermo-electric receiver with the well-known electrolytic type, extending over several months and including some long distance work, showed that in sensitiveness it compared favorably with the latter, being slightly less sensitive on an average than the free Wollaston wire electrode type but more so than the type with the small platinum electrode sealed into glass. The comparisons were made by the shunted telephone method in which the loudness of the signals is measured by the smallness of the shunt *s* (Fig. 2) across the telephones necessary to produce silence. The telephones used were of the ordinary head type and had a resistance of about 1200 ohms.

In the figures *L* represents the inductance, *D* the detector, *E* a

[1] A third form in which the contact metals are tellurium and silicon made up like form one, promises in some respects to be even more satisfactory than the others.

condenser of about 0.05 m.f. used to prevent the thermal currents flowing around through the inductance, T is the telephone and I (Fig. 2) is a choking coil preventing the oscillations flowing through the telephone circuit when the shunt s is closed, B (Fig. 3) is a tuning condenser.

Experiments were also made using 120 cycle alternating currents, various alternating potential differences being applied to the detector from a fall of potential wire. The direct currents were measured by a D'Arsonval galvanometer having a sensitiveness of about 1.10^{-9} amp. per mm. of scale. The results are shown in the following table.

Volts. a.c.	Galvanometer Deflections. mm.
0.015	210
.010	105
.007	50
.005	25
.002	5
.001	1

The resistance of the detector under sensitive conditions varies between 1000 and 3000 ohms therefore the alternating current corresponding to a deflection of 1 mm. lay between 1.10^{-6} and 3.10^{-7} amp.

On account of its high resistance, in working with high frequency currents, the detector is not placed directly in the oscillating circuit but is shunted across the inductance as shown in Figs. 2 and 3.

Washington, D. C.,
January, 1907.

[1] The contact surfaces should be cleaned frequently with petroleum.

UPON THE CHEMICAL NATURE OF THE RADIUM EMANATION.

By L. M. Dennis and L. F. Hawley.

RUTHERFORD, Ramsay and Soddy have described experiments made upon the emanations from thorium and from radium that led them to conclude that if the emanation in either case is due to the presence of a known gas, that gas must be a member of the argon group. Rutherford finally decided,[1] however, that the emanation cannot be due to the excited activity of a member of the argon group or of any other constituent of the atmosphere because when carbon dioxide is passed over thoria, the amount of emanation is the same as when air is substituted for the carbon dioxide.

Notwithstanding this statement by Rutherford it still seemed possible that the emanation might be due to the excitation of an atmospheric gas such as neon, for the reason that it is by no means certain that the carbon dioxide employed by Rutherford was wholly free from atmospheric gases. If he prepared carbon dioxide from marble and an acid, the gas would almost certainly contain appreciable amounts of air. For this reason it seemed desirable to repeat the experiment under such conditions as would exclude the possibility of the presence of a constituent of the atmospheric air, with the aim of ascertaining whether under these circumstances the amount of the emanation is the same as when air is present.

The radioactive material that was employed in the measurement was radium-barium chloride that had been extracted from gummite and uraninite in this laboratory and had been fractionally concentrated by repeated precipitation with hydrochloric acid gas. About 0.1 gram of this material, possessing an activity of 350 uranium units, was dissolved in water. This solution was boiled to free it from air and was cooled in a current of hydrogen.

[1] Radioactivity, p. 226.

The apparatus employed was that shown in Fig. 1. The Kipp generator *T* was first filled with hydrogen and was then charged with pure zinc and with dilute sulphuric acid that had been boiled and cooled in an atmosphere of hydrogen. By means of the connections shown in the drawing an atmosphere of pure hydrogen was always maintained above the acid in the Kipp apparatus. An excess pressure of hydrogen in the upper bulb was automatically relieved by the escape of the gas through mercury in the beaker *H*. Diminished pressure of the gas above the acid was relieved by momentarily opening the stopcock *P*. With the aid of the stopcocks *R* and *S* it was possible to pass hydrogen either directly

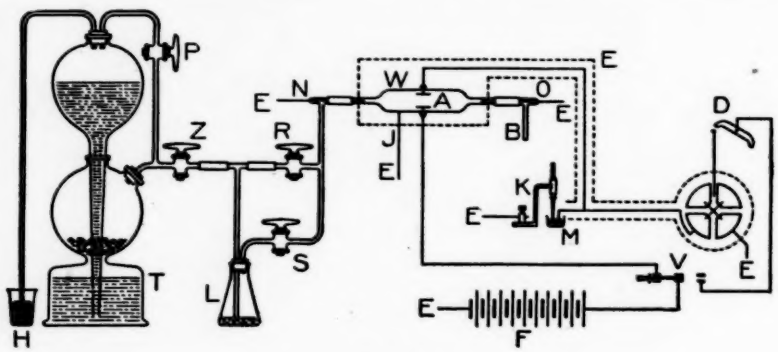

Fig. 1.

through the measuring tube *W* or first through the radium-barium chloride solution in the flask *L* and then through *W*. The measuring tube *W* was of glass, with platinum plates *A* connected to platinum wires fused through the walls of the tube. To diminish the effect of local electrostatic charges on the glass, the outside surface of *W* was covered with tin foil, and the inside surface was coated with silver[1] to within one quarter inch of the platinum wires. These inner and outer metallic layers were grounded by the wires *N*, *O* and *J*. The battery *F* consisted of two hundred small dry-cells, giving a potential of about 190 volts. By means of the switch *V* the battery could be connected either with the lower platinum plate in the measuring tube or with the movable brass bar *D* by means of which the needle of the electrometer was charged. One

[1] Heumann-Kühling, Anleitung zum Experimentieren, p. 697.

pair of quadrants of the Dolezalek electrometer was grounded; the other pair was connected with the upper platinum plate and also with the mercury cup *M* through which it could be grounded by means of the key *K*. The electrometer, the measuring tube, and all the connections between them were screened by brass wire gauze which is represented in the diagram by dotted lines. All parts marked *E* in the diagram are ground connections.

In making the measurements the radium-barium chloride solution was first placed in the flask *L*. The apparatus was then connected up throughout and was allowed to stand for three days. The passage of hydrogen through *L* was then begun and the emanation was led into the tube *W*. After the amount of the emanation had been measured, hydrogen was passed through the solution in *L* until the emanation ceased to be given off. *S* was then closed, *R* was opened and hydrogen was passed through *W* until the emanation was entirely removed from the tube. The exit tube *B* was connected with a flue to prevent the escape of the emanation into the room. The apparatus was then allowed to stand for three days, after which interval it was found that the excited activity on the walls of the tube *W* had entirely disappeared. The measurements were then repeated in the manner described above. After a series of such measurements, the hydrogen generator was disconnected at *Z* and atmospheric air was used instead of hydrogen. The measurements obtained with hydrogen and air are given in the following table, the readings in the table being scale divisions per second.

Hydrogen.	Air.	Hydrogen.	Air.
3.0	8	2.5	9
2.5	10	2.0	11
2.0	11	2.5	8
3.0	9	3.0	10
2.0	12	2.5	10
2.0	10	2.0	11
		Average 2.41	9.9

The above readings may be regarded as indicative of the amount of emanation if one assumes that the conditions, such as sensitiveness of electrometer, proportion of emanation to total gas, etc., were

always the same. The constancy of the readings in both series appears to support the validity of this assumption.

The ratio of the average of the results in hydrogen to the average of the results in air, 0.243, closely approaches the relative conductivities of the two gases for the alpha-ray, 0.226. If the amount of emanation were appreciably greater in air than in hydrogen, the ratio obtained might naturally be expected to vary more widely from that of the relative conductivities of the two gases. Inasmuch as this result was not obtained the above experiments would seem to support the statements of Rutherford that the emanation is not due to the excited activity of a constituent of atmospheric air.

CORNELL UNIVERSITY,
March, 1907.

THE SCIENTIFIC WORK OF DEWITT BRISTOL BRACE.

By Edward L. Nichols.

PROFESSOR BRACE received the degree of doctor of philosophy twenty years ago; at which time he presented to the faculty at Berlin a thesis upon the *Magnetic Rotation of the Plane of Polarization of Light.*[1] His career as a scientific investigator, as indicated by published work, may, however, be said to date from 1897. For twelve years no scientific paper by his pen had appeared, and for this long period he had been isolated in a sense which only those can realize who know the conditions at that time, as regards physics, in the regions lying beyond the Mississippi river. His paper published that year in the Philosophical Magazine under the title: *Observations on Light Propagated in a Dielectric Normal to the Lines of Force*,[2] showed that although silent he had not been idle. The result of this search for an optical effect of the magnetic field upon light moving at right angles to the lines of force was negative. That the effect looked for, if it existed, must necessarily be very small seems to have been fully recognized, and the experiment was planned with this fact in mind. The magnet employed was of unusual size with cores 8″ in diameter and 36″ long, carrying 650 pounds of wire and having a total weight of about two tons. The Faraday effect in glass was eliminated by sending a beam of light through two cylinders each 22 cm. long and placed end to end in the field, and reflecting the beam backward upon its course by a mirror adjusted so as to be parallel with the lines of force. The sensibility of the apparatus was seriously vitiated by depolarization due to accidental double refraction in the glass, for which reason in the final experiments a tube 48 cm. long containing water was employed. This was placed between the poles of the electromagnet, the pole pieces being 50.7 cm. by 8.7

[1] Wiedemann's Annalen, Vol. 26, page 576.

[2] Philosophical Magazine, Series 5, Vol. 44, page 342.

cm. A beam of sunlight was passed four times through the tube by reflection at the mirrors, making a total path of 198 cm. A Lippich half shade polarizer served to polarize the ray in any desired plane with respect to the magnetic field. The plane of polarization of the emerging ray could be determined by means of an analyzer reading to .003° and capable of detecting with certainty changes in the plane of polarization of .01°. No effect could be detected even with this exceedingly sensitive apparatus, indicating entire absence of double refraction within the limits of observation. The conclusion was that, according to Maxwell's view of the state of polarization and stress in such a medium, the pressure at right angles to and the tension along the lines of force affect the propagation of light by an amount less than $2 \times 10^{-14}\lambda$ for a C.G.S. unit of intensity per centimeter.

In 1899 Brace brought out his new spectrophotometer, in which the introduction of an arrangement such as the Lummer-Brodhun body is rendered unnecessary by the very simple and ingenious device of splitting the dispersion prism and silvering a portion of its face so that rays from the two collimators entering the prism in directions symmetrical to the section will issue in parallel paths and give similar spectra suitably located for comparison.

Brace's continued interest in the action of the magnetic field upon light is indicated by his next paper, which appeared in 1901 and which dealt with the resolution of light into its circular components in the Faraday effect.[1] The principle established by Fresnel that two opposite circular vibrations will be equivalent to a linear vibration and *vice versa*, which that investigator had verified for quartz, had been generally accepted as the explanation of the Faraday effect. Direct experimental confirmation of this theory was, however, lacking. It is true that Brace in his thesis work in 1885 had obtained a displacement of the bands when one of the two interfering rays in a Jamin refractometer passed through a cylinder of glass placed between the poles of a magnet, the two rays being circularly polarized. A like result had been obtained by Righi, Becquerel and Cornu. Experiments by the interference method were, how

[1] Philosophical Magazine, Series 6, Vol. I., page 464.

ever, not regarded as conclusive and Brace in this paper undertook to apply the more direct method of refraction.

If natural light is broken up into two opposite circular components when propagated along the lines of force, and if these travel with slightly different velocities, a linear ray moving over a sufficiently long path along the lines of force should be sufficiently separated as to direction to give a double image. By a series of total reflections within a pair of glass prisms placed in the field of a powerful electromagnet, the necessary conditions for resolution were obtained and the existence of the two circular vibrations opposed to each other in direction was established. The phenomnon predicted by Fresnel in his theory and demonstrated by him in the case of quartz was thus found to hold likewise of magnetic rotary polarization. The demonstration in the latter case is of vastly greater experimental difficulty, requiring a very long path through the glass and a powerful field. To observe the separation of the rays the optical conditions must be very carefully preserved and the surfaces at which the successive total reflections occur must be figured with great accuracy. The attainment of a definite result in this experiment is indeed to be regarded as a notable achievement.

At the Denver meeting of the American Association for the Advancement of Science Brace acted as chairman of Section B and in the following year, 1902, he delivered the vice-president's address before that section. He chose for his subject the *Group Velocity and the Wave Velocity of Light* and a more lucid and altogether admirable semi-popular presentation of a difficult subject would be hard to find.

The following year, 1903, marks the opening of the most important, productive and brilliant period of the scientific activity of Professor Brace. In that year he published the first of the series of three papers on his very beautiful and ingenious device, the sensitive strip spectro-polariscope,[1] an instrument which makes it possible to determine the plane of polarization of light with a degree of accuracy hitherto impossible and to detect traces of double refraction unobservable by other means. The second paper dealing with this method of studying polarized light appeared in the PHYSICAL REVIEW

[1] Philosophical Magazine, Series 6, Vol. 2, Page 161.

in 1904, under the title, *Half Shade Elliptical Polarizer and Compensator*.[1] The theory of the instrument was further developed in a paper on the *Errors of the Half Shade Elliptical Polarizer*.[2] The new instrument found immediate application in a series of bold attempts to detect double refraction in matter moving through the ether.[3]

The negative results obtained by Michelson and Morley in their careful experiments upon the interference of light in the direction of the earth's motion and at right angles to the same had led to the suggestion by Fitzgerald and by Lorentz of a mechanical action of the ether upon the refracting medium such as to counterbalance and annul the effect looked for. The medium used in Brace's experiments was water, which was contained in a trough 413 cm. long, 15 cm. wide and 27 cm. deep. A beam of polarized sunlight was sent several times through the entire length of this body of water by a series of reflections, the total path being 756 cm., after which it passed through the sensitive strip and the compensator of the polariscope and through the analyzing nicol. By means of this exceedingly delicate device the slightest trace of double refraction in the medium could have been detected and any such effect due to motion through the ether would show itself in the slight change of optical condition when the trough was placed in the direction of the earth's orbital motion and then at right angles to the same. No such change was observed and when the instrument was set to match in one position and then shifted to 90° no variation of the setting was necessary after the rotation. Tests of the sensitiveness of the method showed that 7.8×10^{-13} represented the greatest difference in velocity or in index between the two components of the ray which could exist without being detected. The effect which might be expected under the theory due to an increase in density in the direction of the motion was computed to be about 1,600 times greater than the smallest effect which could be observed.

Experiments were likewise tried with glass using two prisms cemented end to end and giving a single body 42 cm. long with

[1] PHYSICAL REVIEW, Vol. 18, page 70.
[2] PHYSICAL REVIEW, Vol. 20, page 218.
[3] Philosophical Magazine, Series 6, Vol. 7, page 317.

polished ends. These end surfaces were silvered with the exception of a strip at the top and bottom, and a beam of light from the polarizer was reflected backwards and forwards from ten to eighteen times. The total path through the glass was from 420 to 756 cm. Difficulties with flexure of the trough necessitated, however, a modification of this arrangement in which two cylinders of flat glass each 22.3 cm. long were mounted between the polarizing and analyzing systems. The greatest difference in velocity or in index between the two components which could exist in this glass was found to be

$$4.5 \times 10^{-11},$$

whereas the contraction in the medium from the change computed under Rayleigh's assumptions should have been fifty times greater than that which could have been detected. The conclusion is that the contraction hypothesis cannot explain the negative results of previous experiments and that if the ether does not move with the matter which it surrounds, the effect of the relative motion on the dimensions of a substance must be exceedingly small.

To the problem of the ether drift Professor Brace devoted two further papers, the last to appear before his death. In the first of these[1] he discussed the recent theoretical work of Larmor, Lorentz, Poincaré, Hasenoehrl and Abraham and considered the possibility of the experimental determination of a quiescent or mobile ether. Preliminary tests of one such method, in which he used a very ingenious combination of condensers giving the Kerr effect with the half shade elliptical polarizer, were found to give promise of sufficient delicacy for the direct comparison of the velocities of light with and against the drift.

The second of these[2] important papers is entitled *The Ether Drift and Rotary Polarization.* Mascart, who studied the effect of the motion of the earth on the rotary power of quartz, found that when the direction of the ray is reversed no effect upon the rotary power as large as one twenty thousandth could exist. Rayleigh, who repeated the work in 1902, found that reversal of the ray in the direction of the motion of the earth did not change the rotation

[1] Philosophical Magazine, Series 6, Vol. 10, page 71.
[2] Philosophical Magazine, Series 6, Vol. 10, page 83.

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[1] Philosophical Magazine, Series 6, Vol. 10, page 71.
[2] Philosophical Magazine, Series 6, Vol. 10, page 83.

by one part in a hundred thousand. By substituting oil of caraway seed, which although inferior in rotary power to quartz is free from the optical imperfections of the latter substance, Brace succeeded in showing that the effect of the motion of the earth on the rotation of the plane of polarization in optically active media is probably less than one part in 10,000,000. In some of the measurements the ray of white light was, by means of a suitable series of reflections, sent four times throughout the length of a trough containing the oil thus securing a total path of 1,640 cm. and a total rotation of 21,300 degrees. With the sensitive strip polariscope a direct change of one one hundred and twentieth of a degree could have been detected. The success of this investigation, which pushes the limit beyond which the effect of the earth, if it exist, must lie, probably fifty times as far as had been attained by any previous observer, depends upon the fact that by sending the beam over equal and opposite paths through the liquid complete neutralization of the rotatory effect is obtained whereas the effect due to the earth's motion would be additive.

The experimental difficulties of researches of this character are almost undescribably great and the results obtained by Brace although uniformly negative are to be accounted among the greatest achievements in modern physics. Those of us who have had the pleasure of listening to the presentation of Brace's papers at the American Physical Society and elsewhere are aware that the published results are only a small part of what he had in mind in his attack upon the formidable problem of the ether drift, and there is reason to believe that but for his premature death other and still more striking and important results would have followed.

In addition to the papers already considered in this brief sketch of the scientific work of Professor Brace another important paper which appeared posthumously in the PHYSICAL REVIEW[1] should be mentioned. This paper, which is perhaps the latest and final product of the scientific activity of the author, is on the *Application of Spectral Bands in determining Anomalous Dispersion.* In it the method of determining sudden changes in the index of refraction of a medium by noting the crowding together or the separation of the dark bands

[1] PHYSICAL REVIEW, Vol. 21, page 289.

observed in the spectrum of two systems of interfering rays is fully developed and the advantage of this method with substances such as fuchsine over the usual prismatic method is pointed out. The actual determination of the existence of anomalous dispersion in many cases in which the absorption spectrum indicates its presence is of great importance. This method when properly applied will doubtless add greatly to our quantitative knowledge of the subject.

PUBLICATIONS BY DEWITT B. BRACE.

	Pages.	Figs.
1. "Ueber die magnetische Drehung der Polarisationsebene und einige besondere Fälle der Refraction." (Dissertation, Berlin, 1885).	33	8
2. "On the Transparency of the Ether." (University Studies, 1888).	16	1
3. "Observations on Light Propagated in a Dielectic Normal to the Lines of Force." (Phil. Mag., 1897.)	8	2
4. "Description of a New Spectrophotometer and an Optical Method of Calibration." (Phil. Mag., 1899.)	10	5
5. "On a New System for Spectral Photometric Work." (Astrophys. Journal, 1900.)	18	9
6. "The Observation of the Resolution of Light into its circular components in the Faraday Effect." (Phil. Mag., 1901.)	12	4
7. "The Group Velocity and the Wave Velocity of Light." (Vice-presidential address before Section B of the A.A.A.S., 1902.)	20	6
8. "A Sensitive Strip Polariscope." (Phil. Mag., 1903.)	9	0
9. "A Half Shade Elliptical Polarizer and Compensator." (PHYS. REVIEW, 1904.)	18	0
10. "On Double Refraction in Matter moving through the Æther." (Boftzmann Festschrift and Phil. Mag., 1904.)	12	1
11. "Notes on errors in the Half Shade Elliptical Polarizer." (PHYS. REVIEW, 1904.)	13	0
12. "The Negative Results of Second and Third Order Tests of the 'Æther Drift' and Possibly First Order Methods." (Phil. Mag., 1905.)	10	1
13. "The 'Æther Drift' and Rotary Polarization." (Phil. Mag., 1905.)	14	3
14. "On Application of Spectral Bands in determining Anomalous Dispersion and Achromatic Systems of Various Types." (PHYS. REVIEW, 1905.)		
15. "A Repetition of Fizeau's Experiment on the Rotation of a Refracted Ray Produced by the Earth's Motion." (Phil. Mag., 1905.)		

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE THIRTY-SEVENTH MEETING.

THE Spring meeting of the Physical Society was held in Washington, D. C., on Friday, April 19, and Saturday, April 20, 1907. In the absence of the president, Mr. A. G. Webster was made chairman of the meeting.

The Friday afternoon session was held at the Cosmos Club, and the Saturday sessions at the Bureau of Standards.

The Report of the treasurer for the year 1906 was read by the Secretary and on motion accepted.

Announcement was also made of the resignation of Mr. Hallock as treasurer of the Society, the resignation to take effect on July 1, 1907, and of the election of Mr. J. S. Ames as treasurer for the remainder of the unexpired term.

It was moved, and carried unanimously, that the Society express its thanks to the retiring treasurer, Mr. Hallock, for his long and faithful service to the Society since its founding.

The following papers were presented :

1. An Exact Formula for the Mutual Inductance of Coaxial Solenoids. LOUIS COHEN.

2. The Mutual Inductance of Coaxial Solenoids. E. B. ROSA and LOUIS COHEN.

3. A New Method for the Absolute Measurement of Inductance. E. B. ROSA.

4. Calibration of Calorimetric Platinum Resistance Thermometers. H. C. DICKINSON and E. F. MUELLER.

5. Some New Measurements with the Gas Thermometer. JOHN KAY CLEMENT.

6. An Accurate Calorimeter. W. P. WHITE.

7. Temperature Control of the Cheltenham Magnetic Observatory. JOHN E. BURBANK.

8. Finding Direction by Means of Submarine Sound Signals. (Read by title.) LUCIEN IRA BLAKE.

9. The Heat of Vaporization of Water. A. W. SMITH.

10. The Plan and Object of the Terrestrial Magnetic Work of the Carnegie Institution of Washington. L. A. BAUER.

11. The Influence of Atomic Weight upon the Maxima of Absorption and Reflection Bands. W. W. COBLENTZ.

12. The Fluorescence, Absorption and Magnetic Rotation Spectra of Potassium Vapor. T. S. CARTER.

13. A Redetermination of the Wave-lengths of the Iron Lines. A. H. PFUND.

14. An Apparatus for the Production of a Rotating Electric Arc under Heavy Pressure. W. J. HUMPHREYS.

15. Some Effects of Heavy Pressures on Arc Spectra. W. J. HUMPHREYS.

16. Absorption and Emission Spectra of the Salts of Neodymium and Erbium. J. A. ANDERSON.

17. The Change of Phase due to the Passage of Electromagnetic Waves through Thin Plates and the Index of Refraction of Water for such Waves with Applications to the Optics of Thin Films and Prisms. WM. R. BLAIR.

18. On the Measurement of the Intensity and Damping of Hertzian Waves in Free Air. W. P. WHITE.

19. Lecture Experiment on Magnetically Coupled Systems. A. G. WEBSTER.

20. Mercurous Sulphate, Cadmium Sulphate and the Cadmium Standard Cell. GEO. A. HULETT.

21. A New Method of using the Ultra-violet Microscope. L. J. BRIGGS and W. T. SWINGLE.

22. The Effect of the Time of Passage of a Quantity of Electricity on the Throw of a Ballistic Galvanometer. F. WENNER.

23. The Operation of a Direct Current Shunt Wound Motor in Synchronism with a Tuning Fork. R. H. HOUGH and F. WENNER.

24. (*a*) On the Final Value found for the Ratio of the Electrical Units, v. (*b*) On the Methods Available for the Accurate Determination of v. E. B. ROSA and N. E. DORSEY.

25. The Simultaneous Measurement of the Capacity and Power-factor of Condensers. FREDERICK W. GROVER.

26. On the Variation of Resistances with Atmospheric Humidity. E. B. ROSA and H. D. BABCOCK.

ERNEST MERRITT, *Secretary*.

TREASURER'S REPORT FOR THE YEAR 1906.

Receipts,		
Balance brought forward	$1,742.53	
Interest on balance	69.70	
Annual dues, including exchange, etc.	1,742.30	
Arrears, of annual dues,	41.50	
Initiation fees	51.00	
Life-member fees	100.00	
Total	$3,747.03	$3,747.03

Expenditures,		
Paid the Physical Review	$1,123.50	
Paid Science abstracts	556.00	
Secretary's office	146.90	
Treasurer's office	31.50	
Exchange	4.20	
Printing	54.70	
Total	$1,916.80	$1,916.80
Balance carried forward		1,830.23
		$3,747.03
Normal Income, approximately as above:		
Interest on balance including life fees	$ 75.00	
Annual dues	1,750.00	
Initiation fees, very doubtful, say	30.00	
Total regular income	$1,855.00	$1,855.00
Annual deficit over income		61.00
Regular annual expenditures as above for 1906		$1,916.00

WM. HALLOCK,
Treasurer.

COLUMBIA UNIVERSITY,
April 19, 1907.

THE PHYSICAL NATURE OF METEOR TRAINS.[1]

BY C. C. TROWBRIDGE.

PERSISTENT meteor trains are luminous clouds occasionally formed by meteors, which sometimes remain visible for many minutes. There have been numerous cases where trains have been visible to the naked eye for three-quarters of an hour, and bright enough to be seen two hundred miles distant. The trains are usually ten miles long and gradually expand to about a mile in diameter and are evidently self-luminous.

PREVIOUS INVESTIGATIONS ON THE SUBJECT.

Meteor trains have been observed by many astronomers but are still regarded as an unsolved mystery. But few papers have been published on the subject, other than those which are merely a record of observations.

J. Ennis has described seven different trains, and E. E. Barnard has published a paper relating to the drift of five trains observed at Nashville, U. S. No systematic attempts to collect the records together appear to have been made heretofore, yet it is evident that a large store of useful scientific information concerning the earth's atmosphere lies in these records.

THE SOURCE OF DATA.

The observations of meteor trains have been made chiefly by well-known astronomers at astronomical observatories, who have recorded the

[1] Abstract of a paper presented at the New York meeting of the Physical Society, March 2, 1907.

facts in scientific journals. Most of the records have been since 1866. Some of the astronomers are as follows:

In England: Herschel, Denning, Barkas, Macgregor, etc.

In the United States: Twining, Newton, Young, Barnard, Gilman, etc.

The records are very complete in many cases, in others only a few important items are given. The greater part are trustworthy and scientific observations.

The facts given in this abstract and determined by a comparative study are necessarily of the nature of a preliminary report but they are of sufficient interest to warrant their presentation at this time. About 175 observations have been studied.

Summary of the Facts Determined.

Among the facts determined by this study of meteor trains are the following:

1. Comparisons of drawings and descriptions seem to show that meteor trains often fade first in the center, or that the luminosity is greatest near the outside of the train.

2. The meteor train resembles in a number of respects the phosphorescent afterglow formed in an electrodeless discharge tube, since: (1) The rate of the diffusion of each is of the same order; (2) the afterglow persists at liquid air tempertures; (3) the afterglow under some circumstances persists for nearly 20 minutes; (4) both appear to have line or narrow band spectra.

3. The trains assume curious shapes apparently owing chiefly to air currents at different altitudes in different directions, there being little if any downward movement apparent. The results of a study of the meteor train drifts at great altitudes are given in another paper.

4. *The Duration of Meteor Trains.*

The time that a meteor train remains visible depends largely on the distance between the meteor and the observer. Some idea, however, of the average duration can be had from the following facts. In a list of meteor trains collected, 53 trains having a duration of more than one minute were as follows:

6	remained	visible	over	40	minutes	(40–60 min.)
7	"	"	"	20	"	(20–40 min.)
12	"	"	"	10	"	(10–20 min.)
12	"	"	"	5	"	(5–10 min.)

Thus there were 37 trains which remained visible to the naked eye from five minutes to one hour, the average of 53 trains being 14.8 minutes.

5. *Color of Meteor Trains.*

The observations on the color of meteor trains given are only those where very definite statements regarding the persistent train were made.

Thus the possibility was avoided of confusion of the color of the train proper with the color of the nucleus, or of the mass of sparks which sometimes persists for a few seconds in the track of a meteor.

The colors of the trains observed varied considerably, being red, orange yellow, yellow, green, bluish, silvery and white, but the greater part were green or greenish white. The colors of 25 trains seen at night were as follows:

Red	1
Yellow	1
Green (emerald green and bluish green)	12
Blue (bluish and greenish blue)	4
Silver or grey	4
White	3
	25

In several cases green trains changed gradually to white, and in one case to a dull reddish white. In another a red train changed to bluish.

Thus out of 25 trains, 12 are green and 11 either bluish, silver or white. If, as in several cases observed, the trains change from green to white, a number of white trains are to be expected as a second stage or lower degree of luminosity of an originally green train.

Eleven other trains evidently illuminated by sunlight were colored as follows: Red 2, pink 3, white 3, light blue 1, white changing to red 1, yellow to red 1. These colors are to be expected in the case of trains shining in sunlight.

6. *Altitude of Meteor Trains.*

The observations on the height of meteor trains have been very accurately made in a few cases and approximately made in a number of others. The following are the heights of ten trains that have been computed, the trains having been observed from two or more stations widely apart. The figures below are the number of miles above the surface of the earth: 50, 75, "over 60," 54, 51, 58 (65 to 52), 77, 70, 59, 54. The average of the heights of ten trains given above is 60.8. It is evident that trains seldom if ever occur below 50 miles or over 70, which agrees with a statement of W. F. Denning. In this zone 50 to 70 miles above the surface of the earth. We must expect, therefore, that there are conditions which are favorable to both the formation and the persistence of the mysterious luminosity of the meteor trains. The condition which is most important to electrical discharges and glows in gases is pressure. We may therefore look to the pressure of the atmosphere prevailing at 50 to 65 or 70 miles height as a probable condition which determines the fact that meteor trains are observed at that height only.

7. *Diffusion of Meteor Trains.*

It is evident that the lateral expansion of the train, about one mile in ten minutes, is due mainly to rapid gas diffusion at the low pressure at the altitude of 60 miles. Six trains, three of which the altitude was measured and three estimated at 60 miles, give a mean rate of diffusion of 2.3 meters per second. The diffusion of the afterglow of the electrodeless discharge in small tubes is of the order of several meters per second. Moreover, the calculated diffusion rate of air and a gas like CO_2 at 0.1 mm. gas pressure (which is also the pressure at which the afterglow occurs for long duration), and a temperature of — 150° C. is about 2 meters per second. Therefore, if the meteor train is an afterglow phenomenon it would appear as if the pressure of the atmosphere at 60 miles might be about 0.1 mm. and the temperature not far from — 150° C.

On Atmospheric Currents Above Fifty Miles from the Surface of the Earth.[1]

By C. C. Trowbridge.

THE systematic study of meteor trains is of much importance to meteorology because it is the only means by which atmospheric currents at very great altitudes above the surface of the earth can be studied. The drift of certain trains has been described and discussed to some extent by Newton and Barnard, the latter has described five trains seen in the United States having southeasterly and northeasterly drifts, and also called attention to the importance of this method of studying the movements of the atmosphere.

The writer has made a systematic study of meteor trains chiefly in order to find an explanation of their long-enduring luminosity in the light of recent advances in physics. The investigation has necessitated the making of a catalogue of meteor train observations which has not been done hitherto.

Thus far about sixty-five observations giving meteor train drifts have been found.

The observations when tabulated and studied add considerably to our knowledge of the movements of the atmosphere at great altitudes.

Recently much has been accomplished by cloud observations and by means of kites and balloons provided with recording apparatus, but at far lower heights than those where meteor train movements occur. The limit to which kites can be raised is about three miles, and while cloud observations have been made to a height of eight miles, the usual region for

[1] Abstract of a paper presented at the New York meeting of the Physical Society, March 2, 1907.

cloud study is below this level. In a few cases so-called ice clouds have been observed at a considerably higher altitude.

Balloons with recording meteorological apparatus have been sent to an altitude of about fifteen miles, but the usual limit for successful operation is below twelve miles. The distribution of volcanic dust over the earth's surface by air currents has served as a means of information concerning atmospheric drifts at altitudes at the equator from twenty and perhaps thirty miles, but the only successful studies of importance in this direction are the well known facts of the drifts of the dust of Krakatoa following the eruption of that volcano in 1883 showing a westerly current of 70 to 80 miles per hour at the equator at an altitude probably between 20 and 25 miles above the earth's surface.

Far above these regions lies what may be called the meteor train zone. This extends from 50 to 65 (or perhaps 70) miles above the surface of the earth. Of the hundred or so meteor trains, chiefly observed in England and the United States, in the catalogue made by the writer, in the case of over sixty-five the drift has been observed. Of these sixty-one show the following directions:

Trains seen at night.		Trains seen in daylight or twilight.[1]	
North	9		
Northeast	4		
East	11	Northeast	3
Southeast	12	West	5
South	7		
Southwest	4		
West	0		
Northwest	3		
Drift in two strata in different directions	3		
Total	53		8

The greatest velocities observed were as follows: In one case 125 miles per hour northward drift at over 60 miles altitude above New England in the United States, and in another 61 miles in 30 minutes southeast drift or 122 miles per hour at 54 miles altitude above England.

It is seen from these facts that the drift at an altitude of 50 to 65 miles may be in any direction in northern United States and in England, but that the drift toward the eastward predominates.

In the case of over 25 of the above trains the rate of drift in degrees or miles was recorded.

It has generally been supposed that at great altitudes above the surface of the earth, in the northern hemisphere, there would be a steady eastward drift. The facts determined thus far from a comparative study of

[1] Some evidently below 50 miles.

meteor train drifts clearly show that, at least in the temperate zone, the drift may be in any direction but predominating toward the eastward.

The indications are that there are superimposed currents of different velocities and often in opposite directions in zones of five to ten miles depth; and also that in some cases in one zone there will be a calm, while directly above or below it there will be a current of considerable velocity.

On November 14, 1866, in England and Scotland, there was a current to the south or southeast shown by eight out of ten observations; and on November 14, 1901, the drift in the western part of the United States appeared to be north, while that in the middle and eastern portions was towards the northeast.

Calibration of Calorimetric Platinum Resistance Thermometers.[1]

By H. C. Dickinson and E. F. Mueller.

THE problem was presented of measuring the difference of temperature between the inlet and outlet of a flow calorimeter under conditions requiring thermometers of high sensibility, small size and small time lag. Some form of resistance thermometer seemed to offer the greatest promise of success and after preliminary experiments a pair of such thermometers was built which proved to be so reliable and so adapted to general use that a careful study of them was made. These thermometers consist of four leads of thin sheet copper (.12 mm.), two of them connected to a coil of .02 mm. platinum wire wound on a flat mica strip, and the other two connected together for compensation. The leads and coil are enclosed in a thin copper sheath with mica insulation. This finished sheath is about 1 mm. thick, 7 mm. wide and 15 cm. long, outside dimensions.

The resistance of these thermometers is approximately 30 and 40 ohms in ice and steam respectively, temperature equilibrium is established to $0°.001$ in 3 or 4 seconds and the water equivalent of the thermometers is less than one gram. Comparisons were made with the temperature scale of the Bureau of Standards by means of four primary standard mercurial thermometers at 30°, 40°, 50°, 60°, and 70°. These comparisons showed that the δ of the Calendar formula is constant within the limits of error of the best mercurial thermometer comparisons between 0° and 100°. The values for the two thermometers in question were respectively $\delta = 1.554 \pm .001_5$ and $\delta = 1.556 \pm .002$, representing a probable error (computed) of $0°.0005$ at 50° C. It is therefore shown that these ther-

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 19–20, 1907.

mometers may be used to represent the mean scale of the Bureau of Standards to within 0°.001 between 0° and 100°.

Since this factor δ may be taken as constant it might be equally well determined by observations at a single sufficiently well known point. For the establishment of such a point it was decided to make a redetermination of the transition temperature of sodium sulphate by means of these calibrated resistance thermometers. This temperature was found by Richards and Wells to be 32°.383 C. The use of resistance thermometers offers some decided advantages over a direct determination with mercurial thermometers as it gives a scale about 100 times more open. The results of this investigation are as follows. The transition temperature of a single sample of sodium sulphate is constant to within 0°.001 even for considerable changes of external temperature; some impurities such as sodium chloride may lower this temperature by 0°.001 for .001 per cent. The transition temperature of a sample of "Kahlbaum" sodium sulphate as obtained was found to be 32°.380, and of the purest samples obtained by recrystalization and preparation by different methods gave a temperature between 32°.384 and 32°.385.

ABSORPTION AND EMISSION SPECTRA OF THE SALTS OF NEODYMIUM AND ERBIUM.[1]

BY J. A. ANDERSON.

AS is well known, some of the rare earths give three kinds of spectra, besides those of the arc and spark.

These are:

1. Absorption by solutions.

2. Absorption by solid salts observed when white light is diffusely reflected from them.

3. Emission by solid oxides and phosphates when heated to incandescence.

The object of the present investigation is to try to find some relation between these.

The results show that the absorption spectrum of the solid salts changes with temperature, and that in case of the oxides at high temperature it becomes identical with the emission spectrum, hence showing not only that we are dealing with a pure temperature radiation, but also that the two spectra are due to the same vibrators.

The effect of raising the temperature is, in general, to widen the absorption bands, and a similar change in the absorption of any salt is

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 19–20, 1907.

produced by dissolving it in water. These changes may be explained by assuming that all absorption is caused by electrons inside the metallic atom, the periods of which depend upon the distances and orientation of the other atoms in the molecule. Anything which would increase the amplitude of the swings of these combined atoms, such as a rise in temperature, or the presence of a solvent, would cause a widening of the absorption bands.

JOHNS HOPKINS UNIVERSITY,
April 4, 1907.

THE CHANGE OF PHASE DUE TO THE PASSAGE OF ELECTROMAGNETIC WAVES THROUGH THIN PLATES AND THE INDEX OF REFRACTION OF WATER FOR SUCH WAVES WITH APPLICATIONS TO THE OPTICS OF THIN FILMS AND PRISMS. PART 2.[1]

BY WM. R. BLAIR.

THIS paper deals with the last part of the above subject, the first or experimental part having been presented at the November meeting of the Society in Chicago.

The matter to be presented is as follows:

1. A summary of the experimental results given in Part 1.

2. Theoretical discussion of the change of phase due to the passage of electric waves through thin films and prisms and comparison of computed and observed results.

3. The possible bearing of this work upon the experiments of Johonnot and others on thin films and those of Kundt and Lampa on thin prisms.

RESEARCH OBS.,
MT. WEATHER, VA.

SOME NEW MEASUREMENTS WITH THE GAS THERMOMETER.[1]

BY J. K. CLEMENT.

A NEW gas thermometer has been designed and built in the Geophysical Laboratory, for the measurement of high temperatures, in which a conscientious effort has been made to reduce the magnitude of the errors common to the well-known European gas thermometers. The constant volume principle and the thermoelement as the medium of comparison are retained, but the sensibility of the instrument has been increased until the error of an individual reading is less than the error in

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 19–20, 1907.

the corresponding barometer reading. The bulb (10 per cent. iridium, 90 per cent. platinum, 200 c.c. capacity) is suspended in an air-tight bomb by means of which the pressure outside of the bulb is maintained nearly equal to the inside pressure. This makes it possible to use a high initial pressure — about 300 mm. Hg. — without deforming the bulb at high temperatures, and thus to obtain a sensibility of about 1 mm. of the manometer scale per degree C. without diminishing the range of the instrument.

By taking advantage of the fact that the increased viscosity of the expanding medium (nitrogen) due to rise in temperature affects only the heated portion of the dead space, it has been found possible to reduce the diameter of the cold portion of the capillary tube which connects the bulb with the manometer tube, to 0.5 mm., so that the ratio of the volume of unheated space to the volume of the bulb, $v_a/V = 0.0015$. (Holborn and Day, $v_a/V = 0.0046$; Jaquerod and Perrot, $v_a/V = 0.018$.) The entire correction for the unheated space is therefore only 3.8° at 1,000° and 5.° at 1,200°.

One of the most formidable obstacles encountered was the contamination of the thermocouples with the iridium. To overcome this, it was found necessary to use a heating coil of very pure platinum wire, a nitrogen atmosphere outside the bulb, and to cover the platinum-iridium bulb with a very viscous glaze. Notwithstanding these precautions, the thermocouples deteriorate somewhat when heated for any considerable time at temperatures above 1,000°.

Perhaps the most important source of error which has been overcome is the variation of temperature along the bulb itself due to the cooling at the ends of the furnace tube. By the use of auxiliary heating coils at either end of the main coil, the gradient between the center of the bulb and either end of it has been made less than 0.5°.

We have made a number of comparisons between this instrument and *Pt–PtRh* thermocouples from 300° to 1,200° at intervals of 25°, in which the aggregate error from all sources appears to be well within 1°. Time has not yet permitted a complete series of melting-point determinations, but the following are offered tentatively, subject to minor changes when a sufficient number of determinations shall have been made:

Zinc,	418.4°	Greatest deviation from the mean,					± 0.4°	15 determinations.
Silver,	957.5°	"	"	"	"	"		1 determination.
Gold,	1059.1°	"	"	"	"	"		1 "
Copper,	1080.2°	"	"	"	"	"	± 0.5°	5 determinations.

Some Effects of Heavy Pressures on Arc Spectra.[1]

By W. J. Humphreys.

ARC spectra of a number of metals were obtained at pressures of 42, 69, and 101 atmospheres. A Rowland concave grating of 21½ feet focal length was used, and the work done in the second and third orders of the spectra. The compression cylinder, obtained through a grant from the Rumford Fund, is described elsewhere.

The following are some of the effects noted:

1. Reversals, especially in the shorter wave-lengths, are much more pronounced and frequent under heavy than under light pressures.

2. Pressure seems to increase the widths of all lines, though quite unequally. Occasionally lines are found narrower when taken under heavy pressure, but apparently this is due to decrease in exposure, since in these cases the lines are rather weak with gradually fading edges.

3. The lines of the carbon and cyanogen bands are not certainly displaced even at the highest pressures.

4. The wave-lengths of all other lines examined increase approximately proportional to the increase in pressure up to the highest pressures used, though this increase or shift is very different for different lines even of the same element.

5. A number of these shifts amount to approximately half an Ångstrom unit, and a few are displaced to even a distinctly greater extent.

6. There is to me no convincing evidence that any considerable portion of the shift is due to anything other than the *total* pressure.

7. The extent of the shift of a line is practically independent of the amount of material used, and whether or not it is reversed.

8. So far as I can judge from the scanty data on the Zeeman effect, those lines which are strongly separated by a magnetic field have correspondingly large pressure shifts, and conversely.

9. The increase in wave-length is only a small fraction of what it would be if the luminous particles obeyed the same laws that control a Hertzian oscillator.

10. The intensities of titanium lines seem to be invariably increased by pressure; while iron, in this particular, behaves very irregularly. Some of its lines are increased in intensity, others are decreased, and a number even disappear.

The many spectrum negatives used in this work were obtained by the aid of the large spectroscope and the pump of the liquid air plant of the University of Virginia, and I wish to thank President Alderman and Professor Smith for their kindness in placing this excellent equipment at my disposal.

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 19–20, 1907.

AN APPARATUS FOR THE PRODUCTION OF A ROTATING ELECTRIC ARC UNDER HEAVY PRESSURE.[1]

BY W. J. HUMPHREYS.

IN the production of arc spectra under heavy pressure it is necessary for one of the poles to rotate, since it is not possible at pressures of fifty or more atmospheres, at least with ordinary voltages of two hundred and fifty or less, to maintain a steady arc between fixed poles.

The apparatus that proved efficient for this purpose consists of a cylindrical forged steel shell, with spherical ends terminating in short extensions through which the carbon carriers pass, and onto which attachments for tightening the packing around these carriers and for moving them endwise are screwed. The inside diameter of the shell, which is one and one half inches thick, is six inches, and its length sixteen inches. One side of the shell carries, at its middle, a thick tube onto the end of which is screwed a cap containing a quartz window cemented in place. A steel tube, carrying a pressure gauge and leading through a moisture separator provided with a stopcock to a compression pump, is screwed into the shell near one end. The carbon carriers, heavy steel tubes, pass through stuffing boxes at the ends of the shell and are easily adjusted lengthwise by means of wheel-shaped nuts fitting on screw threads cut some four or five inches along their outer ends. The tube that carries the fixed pole has a rod for the current running along it axially and screwed at the outer end through a piece of ebonite held in place by a suitable cap, and passing at the inner end through a block of mica. The space between the rod and the tube is tightly packed with ground mica, which produces both an airtight condition and electric insulation. The rotating pole, excentrically touched by the fixed pole, is a disk of carbon as large as will pass through the packing of the stuffing box — about an inch in diameter — suitably mounted on the end of a rod running through a steel tube similar to the one carrying the fixed pole. The inner end of this tube has an iron plug through which the rod passes in a loose bearing. The outer end is provided with a cap through a part of which the rod passes with a tolerably close fit. The end thrust of this rod is supported by a ball bearing, and a grooved pulley is provided near this bearing for rotation. Appreciable air leak is prevented by packing the space between the rotating rod and steel tube with Ceylon graphite, which is micaceous in structure, and readily blown into any leaky opening. It is also a lubricant and a conductor, both desirable qualities in this case.

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 19–20, 1907.

The apparatus worked nicely at all pressures tried, the highest being one hundred and one atmospheres.

I wish to thank the committee of the Rumford Fund, American Academy of Sciences, for a grant that met most of the expense incident to the construction of the above piece of pressure apparatus.

The Operation of a Direct Current Shunt Wound Motor in Synchronism with a Tuning Fork.[1]

By R. H. Hough and F. Wenner.

THE work here reported was undertaken as a first step in a problem in alternating currents where it is necessary to have a very constant frequency even when the power is changed by as much as 1 or 2 k. w. Several different machines were used in this work but the one used most was a 5 k. w. Westinghouse converter. A machine sometimes used in connection with the converter was a ½ h. p. Edison motor. The converter was generally used as a motor though at times it was used either as a dynamo or an inverted converter.

Let us consider the case where the regulation is effected through the dynamo. On the shaft of the motor is a half ring commutator. Two brushes bearing on this commutator are connected to the terminals of an auxiliary resistance in the field circuit of the dynamo. This arrangement shunts the auxiliary resistance of the dynamo during one half of each revolution of the motor armature. This increases the field current of the dynamo and therefore the voltage generated. In parallel with these brushes is a contact operated by the tuning fork and closed during one half of each of its vibrations.

When the phase of the commutator is 90° behind the phase of the tuning fork the auxiliary resistance is shunted during three fourths of each revolution of the motor armature and the dynamo generates a definite voltage corresponding to this phase difference. Now if due to any cause whatever the speed of the motor is decreased slightly, the phase difference continually increases (and therefore the field current of the dynamo and consequently the voltage generated) until the power supplied to the motor is sufficient to keep it from falling below sychronism. Or if due to any cause whatever the speed of the motor increases slightly the phase difference continually decreases and consequently the power supplied to it. This prevents it from going above synchronism.

In the above arrangement which is one of the several used the dynamo voltage is so regulated as to keep the motor at a constant speed. When the motor is not loaded the dynamo generates a constant voltage even when its speed or the load upon it is changed.

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 19–20, 1907.

THE FLUORESCENCE, ABSORPTION, AND MAGNETIC ROTATION SPECTRA OF POTASSIUM VAPOR.[1]

BY T. S. CARTER.

THE red fluorescence, which is exhibited by the vapor of metallic potassium when it is illuminated with a beam of white light, was first observed by Wiedmann and Schmidt.[2]

The metal was enclosed in a glass bulb which was evacuated and sealed. The bulb was heated until the metal was vaporized, and upon concentrating a beam of arc or sunlight upon the vapor, an intensely red cone of fluorescent light was observed. Examined with a spectroscope the spectrum showed a band in the red whose position was given by the observers as 695–615.

In view of the remarkable properties which have been shown by the vapor of metallic sodium, it was thought very desirable to take up the study of potassium vapor along the same lines.

The vapor has been studied from the standpoint of its absorption, its fluorescence when excited by white light and its magnetic rotation of the plane of polarization. Photographs of all these spectra have been obtained.

One of the most interesting facts brought out by these investigations is the almost exact similarity between these spectra and the corresponding spectra which are shown in the case of sodium vapor.

The vapor was investigated in glass bulbs during the earlier experiments, but it was found that it could be studied to much greater advantage in a highly exhausted steel tube. The arrangement of apparatus was the same as that made use of in the work on sodium vapor.[3]

All the photographs were taken with a grating spectrograph containing lenses of long focal length. The exposures in the case of the fluorescence spectra were from two to three hours. About fifty lines could be measured upon the dividing engine and their wave-lengths determined by comparison with the iron arc. These lines extend from wave-length 6,300 to 6,770.

The absorption spectrum observed with the vapor in a sufficiently dense condition consists of an immense number of very fine, sharp lines. The wave-lengths of the principal lines, which extend from 6,295 to 6,860, were determined.

When the negatives of the fluorescence and absorption spectra are superimposed the principal dark emission lines of the former coincide

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 19–20, 1907.

[2] Wied. Ann., 57, 477, 1896.

[3] The Fluorescence of Sodium Vapor, Phil. Mag., Nov., 1905, R. W. Wood.

with the light absorption bands of the latter throughout the plates. Although there are a great many more lines in the absorption spectrum.

The magnetic rotation spectrum which is shown when polarized light is passed in succession through magnetized potassium vapor and a mica prism, set originally for extinction, has also been photographed with the same apparatus. The magnetic rotation lines are very sharp. The spectrum contains about eight very strong lines which form a series, the lines being nearly equidistant in the spectrum. These lines are represented in the fluorescence spectrum and are due to the very strong absorption lines. In all about twenty-five lines were measured up.

THE MUTUAL INDUCTANCE OF COAXIAL SOLENOIDS.

BY E. B. ROSA AND LOUIS COHEN.

VARIOUS formulæ for the calculation of the mutual inductance of coaxial solenoids have been given from time to time. Although few of these formulæ are exact, several of the approximate formulæ permit inductances to be calculated with very great accuracy by using a sufficient number of terms of the series by which they are expressed. We have collected a number of these formulæ, extended some of them to improve the accuracy, and have compared and tested them by numerical calculations. The following are the more important of the formulæ:

1. Maxwell's formula (extended) for coaxial solenoids of equal length, the inner having a radius considerably smaller than the outer, the length being several times the diameter for accurate results.

2. Roiti's formula (extended) for coaxial coils, the inner being considerably shorter than the outer. This is a very convenient and accurate formula. The proof is given in the complete paper for the first time.

3. Gray's formula for coaxial coils the length being $\sqrt{3}$ times the radius for both coils.

4. Searle and Airey's formula for coaxial coils, the length of the inner being considerably shorter than the outer.

5. Cohen's formula for concentric coaxial coils in general, which is used as a test for some of the others.

The numerical examples given show what degree of accuracy is attainable, and what form of coils can be computed by the various formulæ. The full paper will shortly be published in the Bulletin of the Bureau of Standards.

[1] Abstract of a paper presented at the Washington meeting of the Physical Society April 19–20, 1907.

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ERRATA.

PAGE 310, seventh line from bottom omit the word "*find.*"

PAGE 317, eighth line from the top read "*maximum*" for "*minimum.*"

PAGE 318, fourth line from top read "*curve, c,*" for "*curve, b,*" and *transmission curve, b,*" for "*transmission curve c*"; eighth line from top and fifth line from bottom read "*curves a, b, c,*" for "*curves b, c, d.*" Curve *d* in Fig. 7 is curve *c* in Fig. 6.

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